

Negotiations with Limited Specificifiability*

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Abstract

This paper studies negotiations with *limited specificifiability* —each of the participating parties may not be able to fully specify a negotiation outcome. We construct a class of negotiation protocols under which we can conduct comparative statics on specificifiability as well as move structures. We find that asynchronicity of proposal announcements narrows down the equilibrium payoff set, in particular leading to a unique prediction in negotiations with a “common interest” alternative. The equilibrium payoff set is not a singleton in general, contains any payoff profile that gives each player no less than her worst Pareto-efficient payoff, and is larger under limited specificifiability than under unlimited specificifiability. The degree to which limitation on specificifiability affects the prediction of a negotiation depends also on the fine details of how such limitation is imposed.

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1 Introduction

Negotiations pervade in our social, economic and political lives. They take place in the contexts of labor union, legislature, mergers and acquisitions, climate change, disarmament, international trade, and so forth. This paper introduces a novel concept *limited specifiability* in negotiations, and analyzes its effect on negotiation outcomes. Specifiability refers to the degree to which each participant of a negotiation can specify an outcome in their proposals. It is limited if one cannot fully specify any of the exact outcome, and is unlimited if one can do so for each of possible outcomes.

Limited specifiability is abundant in real negotiations. In negotiations among different countries, say the Conference of the Parties (COP) meetings for climate change, the representative of each country may not be able to make a proposal that goes against the benefit of a certain influential interest group in her own country. It may be the case that each of the negotiating parties has their own exclusive right to change a certain aspect of negotiation outcomes. Two firms may be quoting prices of their products until they settle down. Communication between the firms are not usually allowed, but in practice there would be various ways to imperfectly convey reactions to the opponent firm, and such a situation resembles negotiations with limited specifiability (firms cannot specify a price profile but only their own price). The design of an online market may be such that sellers (buyers) can only specify their minimum (maximum) acceptable prices.

Some of these situations are more complicated than others, and sometimes specifiability may vary across time or histories of past proposals and responses. As a first study of considering various cases like those, we focus on analyzing the effect of specifiability by fixing the degree of specifiability constant over time, and examine when and how different specifiability conditions lead to different outcomes.

In our model, there is a set of alternatives X and each player i is associated with a set of proposals \mathcal{P}_i , which is a collection of subsets of X . When a player moves, she expresses a Yes/No response to past proposals and make a counter-proposal from \mathcal{P}_i . We say that player i 's specifiability is limited if she does not have a singleton set (of an alternative) in \mathcal{P}_i , and it is unlimited if she has all the singleton sets in \mathcal{P}_i . Once players reach a consensus on an alternative (we will explicitly define the meaning of consensus), then players obtain corresponding payoffs from that alternative. If the negotiation continues indefinitely without reaching a consensus, then players receive

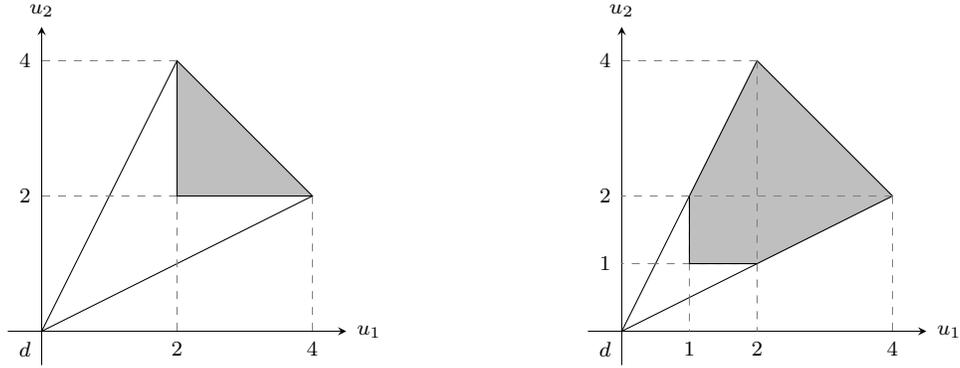


Figure 1: The SPE payoff sets under unlimited speciifiability (left) and limited speciifiability (right).

pre-determined disagreement payoffs. We analyze a subgame perfect equilibrium, which we show exists, of this negotiation game.

We find that the timing of making proposals affects the comparison of possible outcomes under different speciifiability conditions. Specifically, when players' proposals are made in a synchronous manner, we obtain a “folk theorem”— all payoff profiles no worse than the disagreement payoffs are achievable in subgame perfect equilibrium under arbitrary speciifiability conditions. On the other hand, if moves are asynchronous, the equilibrium payoff sets are smaller under both specification conditions. In particular, when there exists an alternative that Pareto-dominates all other alternatives no worse than the disagreement payoffs, then it is the unique outcome of the negotiation game. In general, the equilibrium payoff set is not a singleton, and is smaller under unlimited speciifiability than under limited speciifiability. The main reasons for these results are that asynchronicity helps players commit to realizations of final outcomes, and the commitment powers vary across different speciifiability conditions. In particular, limited speciifiability implies that each proposal entails a smaller degree of commitment to a final outcome, and hence leaves more scope for punishment conditional on deviations. This leads to a larger set of equilibrium payoffs than in the case under unlimited speciifiability.

Negotiations under limited speciifiability and the ones under unlimited speciifiability are quite different. If we further add variations of negotiation protocols in terms of the timing of making proposals and try to examine the effect of such variations on the difference of the negotiation outcomes under different speciifiability conditions, we need to consider quite a large class of negotiations that at least superficially look very

different from each other. Thus, in order for our comparison of different negotiation protocols to make sense, we need a single coherent framework with which we can study a sufficiently wide class of negotiation protocols. For this purpose, we define a negotiation protocol as a collection of three rules— a *proposer rule*, a *specification rule*, and a *termination rule*. Roughly, a proposer rule determines who speaks when, a specification rule designates the collection of proposals each player can announce, and a termination rule determines the histories under which a negotiation terminates and the outcome associated with such a termination. The idea is to vary only one rule in conducting comparative statics, holding fixed the other two.

The heart of this exercise is to define a termination rule solely as a function of histories. This in particular enables us to isolate specification rules from a termination rule: in other words, we can meaningfully compare two specification rules under a truly single termination rule. In order to define a sensible termination rule, we first define what it means for player i to be *ok* with an alternative x given a history of proposals and responses, and then consider a termination rule such that once all players are ok with x at a given history, then the negotiation ends with x at that history. Briefly, player i is ok with x under a history if her announcement at that history gives rise to the unique intersection $\{x\}$ with the latest proposals by the opponents after which no player announces No. We call such a termination rule *consensual*. The comparison of outcomes under different specifiability conditions are conducted under the consensual termination rule.

Under two-player asynchronous proposer rules, we show that the SPE payoff set is larger under limited specifiability than under unlimited specifiability. It turns out that each of the additional payoff profiles achievable under limited specifiability is always Pareto-dominated by some payoff profile that is already achievable under unlimited specifiability. One may question the importance of examining these payoffs. To understand its importance, we consider the case in which knowledge of rationality is of only a finite order, and show that the set of SPE payoffs under each specifiability is the same as the set of achievable payoffs under the same specifiability condition with only a few order of knowledge of rationality (modulo a technical assumption). In particular, this implies that the additional payoffs achievable under limited specifiability are not achievable under unlimited case even when there is a great deal of strategic uncertainty regarding the future play, while those payoffs represent the worst-case payoffs that players can expect when they do not know if their future play resembles

equilibrium but only know a few orders of knowledge of rationality. In the applications that we have in mind, it is unrealistic to relate the key determinants of negotiation outcomes solely to impatience¹ —for example, a COP conference would continue during a fixed short period of time, and the stake of the negotiation is so large and long-term that the discounting that would take place during the course of the negotiation period would not affect the outcome.² Examining the structure of the equilibrium strategies achieving additional payoffs is also helpful in understanding the key tradeoff: such strategies involve punishment, and the scope for such punishment depends on the commitment powers that differ under different specification rules.

The paper is organized as follows. The rest of this section discusses the related literature. Section 2 formulates our model of negotiations, by defining proposer, specification, and termination rules. In particular, we define the consensual termination rule. We start with benchmark analysis in Section 3, where we prove the “folk theorem” under synchronous moves. In Section 4, we analyze the properties of the equilibrium outcomes with asynchronous moves which are independent of specifiability conditions. Section 5 discusses different predictions under limited and unlimited specification rules. Section 6 discusses further topics, such as strategic uncertainty, the effect of discounting, and stochastic announcements. Section 7 concludes. It also discusses a demonstration of a wide class of negotiation protocols that our model nests. All the proofs are relegated to the Appendix.

1.1 Literature Review

Bargaining

Various models of bargaining have been proposed in the literature, including ones with synchronous proposals and others with asynchronous proposals. Models that originate from the Rubinstein-Ståhl bargaining model (Rubinstein (1982) and Ståhl (1972)) usually assume asynchronicity, while some other models assume synchronicity. For example, the Nash demand game (Nash, 1953) is a synchronous one-shot game.

Commitment Power under Synchronous vs. Asynchronous Moves

¹The discrepancy between the SPE payoff sets under the two specifiability conditions is nuanced in the presence of discounting. See Section B.4 of the Online Appendix.

²Crawford (1990) questions the overfocus on impatience in the bargaining literature and argues that strategic uncertainty might be a key determinant of bargaining outcomes.

Our paper is related to the various strands of dynamic game literature which examine the idea that asynchronicity narrows down the equilibrium payoff set. Maskin and Tirole (1987, 1988a, 1988b) study the effect of the timing structure of their oligopoly competition model. In the repeated-games literature, Lagunoff and Matsui (1997) show that if all players have the same payoff function then there is a unique SPE outcome.³ Caruana and Einav (2008) show a uniqueness result under asynchronicity and switching costs in their finite-horizon model. Calcagno et al. (2014) also examine the effect of asynchronicity in a type of finite-horizon games and provide selection results under a setting with stochastic opportunities over finite horizon. The general idea behind these results is that player i 's action a_i at time t determines her action at $t + 1$ under asynchronicity, so i can guarantee the payoff from (a_i, a_{-i}) such that a_{-i} is part of the supergame strategy satisfying a best response condition. Our point is that the power of such commitment may be nuanced by the possibility of punishments in negotiation games, and may change depending on specificity.

Cheap Talk and Pre-game Communication —how we should model

In the literature of cheap-talk pre-game communication with complete information such as Farrell (1987, 1988) and Rabin (1994), the role of cheap-talk communication has been studied as a device for players to convey their intentions for their decisions.⁴ One problem endemic in the literature is that, modeling a negotiation/communication process is difficult as Farrell (1988) puts it: “there are no obviously ‘right’ rules about who speaks when, what he may say, and when discussion ends.” Our formulation of negotiation protocols using three rules makes it possible to compare equilibrium outcomes under different negotiation protocols.

Specifically, consider the case in which the set of alternatives corresponds to the set of action profiles of a normal-form game. Some models in the literature assume that players can announce action profiles of the underlying game, while others assume that they can only announce their own actions. The move structure also varies across models. For example, Farrell (1987), Kalai (1981), and Rabin (1994) assume synchronous moves while Santos (2000) assumes asynchronous moves. These models are quite different from each other and hence it is difficult to meaningfully compare

³See also Yoon (2001), Lagunoff and Matsui (2001) and Dutta (1995) for conditions on folk theorems in asynchronous repeated games.

⁴See also a survey by Farrell and Rabin (1996). Crawford (1998) surveys experiments on cheap-talk communication.

their results to understand the effects of limited specificity and/or move structures. Our model deals with unlimited/limited specificity and synchronous/asynchronous moves in a unified framework to make comparison possible.

Cheap Talk and Pre-game Communication —what players achieve

We study relationships between the outcomes of negotiations and the structure of the underlying game. In cheap-talk models, however, there exists a babbling equilibrium unless an assumption is imposed on the relationship between talk and choice of actions. So certain relationships need to be assumed from the outset. For example, the models of Farrell (1987), Rabin (1994), and Santos (2000) assume that a Nash-equilibrium play of the underlying game is carried out after the communication phase, and thus these models do not address whether we should expect Nash equilibrium after communication. The environment we study, on the other hand, is the one where players can bind their actions of the underlying game during the course of the negotiation. This allows us to study how the set of outcomes of the negotiation game is related to that of Nash equilibria in the given underlying game. It turns out that these two sets can be disjoint in general, while there are certain relationships in special cases (e.g., if the underlying game has a unique Pareto-efficient action profile, then it is a unique SPE outcome and is in the set of Nash equilibria).

Also, whether outcomes of bargaining/communication games are restricted to Pareto-efficient outcomes has been studied in the literature (see Crawford (1998) and Farrell and Rabin (1996)). This question has attracted considerable attention especially when the underlying normal-form game has a unique Pareto-efficient action profile (see Farrell (1988), Rabin (1994), and Santos (2000)). Although these papers analyze quite different sets of questions than ours, their results and ours are similar in that the equilibrium outcomes may not be Pareto efficient in general, while if there is a unique Pareto-efficient alternative then it is a unique equilibrium outcome.

Revocable Pricing and Asynchronicity in Oligopoly

One important feature of our negotiation game is that announcements of proposals are revocable before the parties ultimately agree on a certain alternative. In the industrial-organizations literature, revocable pricing is studied to explain “kinked demand curves,” for example, by Bhaskar (1989), Farm and Weibull (1987), and Stahl (1986). Especially, Bhaskar (1989) studies a game which he calls the quick-

response game where (i) two firms in a Bertrand duopoly sequentially announce their prices; (ii) they can change their price announcements in reply to their opponents' announcements; and (iii) they are bounded to take their announced prices once there is a firm that repeats the same price in a row.⁵ In that model, he shows that the two firms can sustain the monopoly price in a unique equilibrium. If we formulate Bhaskar's (1989) termination rule within our framework and apply it to a general class of underlying games, we obtain a different set of equilibrium payoffs. We discuss the difference in more detail in Appendix B.9. Our framework can also nest the models of Farm and Weibull (1987) and Stahl (1986).

2 The Model

Component Games. Consider a tuple $G = \langle N, X, (u_i)_{i \in N} \rangle$. The set of players $N := \{1, 2, \dots, n\}$ is finite (with $n \geq 2$). The set of *alternatives*, X , is a (non-empty) metric space. Player i 's (vNM) payoff function is $u_i : X \rightarrow \mathbb{R}$. We call G a *component game*. Throughout, we treat a generic player i as female.

Negotiation Games. Given G , the set of players N engage in rounds of negotiations, which we call a *negotiation game* (or simply a *negotiation*) of G . In the negotiation, the players make announcements in a given order, where each player's announcement comprises of a subset of alternatives, referred to as a *proposal*, and a *response* to the opponents' previous proposals. The payoffs from an agreed-upon alternative in the negotiation game is defined as those from the component game. If the players do not agree on any alternative (i.e., the negotiation lasts indefinitely), players obtain the *disagreement payoffs* $d \in \mathbb{R}^n$.

Formally, the negotiation game of G is an extensive-form game denoted by $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi \rangle$, where G is the component game, $d \in \mathbb{R}^n$ is the vector of disagreement payoffs, ρ is the *proposer rule*, $(\mathcal{P}_i)_{i \in N}$ is the *specification rule*, and φ is the *termination rule*. The proposer rule determines who can speak when. The specification rule designates what each player can potentially announce at each time. The termination rule determines when the players conclude their negotiation. We will formally explain each of these components in what follows.

⁵There are no Yes/No responses in his model.

Histories. We first define the set \mathcal{H}^* of possible histories that can occur during the course of *any* negotiation: negotiations take place over time $t \in \mathbb{N}$ in the discrete time setting, and we specify the history at time “0.” Given the set \mathcal{H}^* , the three rules, proposer, specification, and termination rules, determine the set $H(\subseteq \mathcal{H}^*)$ of histories and the set $Z(\subseteq H)$ of terminal histories of the negotiation game Γ .

We assume the situation is as if the negotiation starts after everyone says (No, X) in period 0. That is, we let the initial history be $h^0 := ((N, ((\text{No}, X))_{i \in N}))$. Let $\mathcal{H}^0 := \{h^0\}$.⁶

The set $\mathcal{H}^* := \bigcup_{t \in \mathbb{N} \cup \{0, \infty\}} \mathcal{H}^t$ of possible histories comprises of sequences of a set of past announcements together with the identities of the speakers: for each $t \in \mathbb{N} \cup \{\infty\}$, we let \mathcal{H}^t be the set of histories of the form $((I^{t'}, ((R_i^{t'}, P_i^{t'}))_{i \in I^{t'}}))_{t'=0}^t$, where $I^{t'} \in 2^N$, $R_i^{t'} \in \{\text{Yes}, \text{No}\}$, and $P_i^{t'} \in \mathcal{P}_i$ for each $i \in I^{t'}$ and $t' \in \{t'' \in \mathbb{N} \mid t'' \leq t\}$ (and $I^0 = N$ and $(R_i^0, P_i^0) = (\text{No}, X)$ for each $i \in N$). While $R_i^{t'}$ stands for player i 's response at period t' , $P_i^{t'}$ stands for player i 's proposal at period t' . While each player can potentially announce a subset of X as their proposals, they can, at the same time, respond to the opponents' previous proposals by saying Yes or No.⁷

Let $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. For each history $h \in \mathcal{H} := \bigcup_{t \in \mathbb{N}_0} \mathcal{H}^t$ with a finite length, the time associated with history h is denoted by $t(h)$ and h can be written as

$$h = \left(\left(I^{t'}(h), ((R_i^{t'}(h), P_i^{t'}(h)))_{i \in I^{t'}(h)} \right) \right)_{t'=0}^{t(h)}.$$

Also, for such h and $t' \in \{0, \dots, t(h)\}$, we denote by $h^{t'}$ the subhistory of the following form:

$$h^{t'} := \left(\left(I^{t''}(h), ((R_i^{t''}(h), P_i^{t''}(h)))_{i \in I^{t''}(h)} \right) \right)_{t''=0}^{t'}.$$

We denote $h' \sqsubseteq h$ ($h \sqsupseteq h'$) if h' is a subhistory of h , i.e., $h' = h^{t'}$ for some $t' \in \{0, \dots, t(h)\}$. We also denote $h' \sqsubset h$ ($h \sqsupset h'$) if h' is a proper subhistory of h , i.e., $h' = h^{t'}$ for some $t' \in \{0, \dots, t(h) - 1\}$.

Proposer Rules. A pre-determined function, which we call a proposer rule, deterministically assigns the *proposers* or *speakers* who can make announcements after

⁶We will often omit to write h^0 as part of a longer history.

⁷The assumption that a player can announce (Yes, \cdot) even in the period before which no one has spoken may not look natural. However, under the consensual termination rule that we define below, a player's response Yes or No in such a period has no consequence on the set of equilibrium payoffs.

each possible history. That is, the proposer rule is a function $\rho : \mathcal{H} \rightarrow 2^N$.⁸ We assume that for any $h \in \mathcal{H}$, there exists $t' \in \mathbb{N}$ such that, for any $h' \in \mathcal{H}$ such that (i) $t(h') = t'$ and (ii) $h' \sqsupset h$, we have $N = \bigcup_{h'' \in \mathcal{H}: h \sqsubseteq h'' \sqsubseteq h'} \rho(h'')$. In words, for any history $h \in \mathcal{H}$ with finite length, there is a time $t' (\geq t(h))$ such that, for any proper superhistory h' of h with time length $t' = t(h')$, every player has an opportunity to speak between the histories h and h' . A proposer rule is said to be *synchronous* if $\rho(\cdot) \in \{\emptyset, N\}$. The proposer rule is said to be *asynchronous* if $|\rho(\cdot)| \leq 1$. By abusing notation, we often denote by $\rho(h) = i$ when $\rho(h)$ is a singleton $\{i\}$.

Specification Rules. A specification rule $(\mathcal{P}_i)_{i \in N}$ designates a collection of subsets of alternatives that each player can potentially propose.⁹ We assume that, for each $x \in X$, there exists a profile $(P_i)_{i \in N}$ of proposals with $P_i \in \mathcal{P}_i$ for each $i \in N$ such that $\bigcap_{i \in N} P_i = \{x\}$.¹⁰ This assumption says that players can collectively agree on an alternative x .¹¹ Note that we also allow each player i to announce (No, P_i) after a history h , in reply to the opponents' previous announcements, in which her last proposal under h coincides with P_i . In this case, her announcement (No, P_i) can be interpreted as the message that she is not satisfied with the opponents' previous announcements and yet her proposal is P_i .

Player i 's specification rule \mathcal{P}_i is said to be *unlimited* if $\{x\} \in \mathcal{P}_i$ for all $x \in X$. It is *limited* if $\{x\} \notin \mathcal{P}_i$ for all $x \in X$. If player i 's specification rule is unlimited, she can specify a single alternative in her proposal. If it is limited, on the other hand, she cannot specify a single alternative but she can only specify a set of alternatives. We will simply say that specification is unlimited (limited) if it is so for all players.

⁸This specification allows for the possibility of $\rho(h) = \emptyset$, in which no player can make her announcement after the history $h \in \mathcal{H}$. See Section B.6 for its potential role.

⁹To make our point as clear as possible, we assume that there is a history-independent set $\mathcal{P}_i \subseteq 2^X$ such that after each history, each player who makes a proposal chooses it from \mathcal{P}_i .

¹⁰Although unnecessary, one might be able to add some set-algebraic properties on \mathcal{P}_i such as closure of \mathcal{P}_i under (finite/infinite) union and intersection and complementation. For example, if player i is able to propose $P_i^1, P_i^2 \in \mathcal{P}_i$, we could assume that she is also able to propose its union and intersection, i.e., $P_i^1 \cup P_i^2, P_i^1 \cap P_i^2 \in \mathcal{P}_i$. We can also assume that $\emptyset \in \mathcal{P}_i$ for each player i . Such a possibility may be important in real negotiations, because one may withhold making proposals until the other parties make proposals.

¹¹This assumption is not restrictive in the following sense. Consider the subset Y of alternatives on which players can collectively agree, i.e., $Y := \{x \in X \mid \text{there is } (P_i)_{i \in I} \text{ such that } \{x\} = \bigcap_{i \in I} P_i\}$. In real negotiations, Y might be a proper subset of X due to some (e.g., budget or technological) constraints. As long as Y is not empty, however, we can redefine each player's specification rule as $\bar{\mathcal{P}}_i = \{P_i \cap Y \in 2^Y \mid P_i \in \mathcal{P}_i\}$ so that for every $x \in Y$ there is a profile of proposals $(\bar{P}_i)_{i \in I}$ with $\{x\} = \bigcap_{i \in I} \bar{P}_i$.

Termination Rules. A termination rule φ is a function $\varphi : \mathcal{H} \rightarrow X \cup \{\text{Continue}\}$, which determines, for each history $h \in \mathcal{H}$, whether the negotiation ends with a certain alternative $x \in X$ at that history or it continues, conditional on the negotiation having continued up to that history.

We define a particular termination rule which we call the *consensual* termination rule $\varphi^{\text{con}} : \mathcal{H} \rightarrow X \cup \{\text{Continue}\}$: in order for $\varphi^{\text{con}}(h) = x$, it requires that every player i is *ok* with the alternative x at the end of the history h .

In order to define what it means by a player being ok with an alternative, we first define, for any given history h , two critical times of a negotiation under the history h . First, for each $h \in \mathcal{H}$ and $j \in N$, let $t_j^{\text{sp}}(h)$ be the latest time at which j has spoken until period $t(h)$. Namely, $t_j^{\text{sp}}(h) := \max\{\tau \in \mathbb{N}_0 \mid \tau \leq t(h) \text{ and } j \in I^\tau(h)\}$. Second, for each $h \in \mathcal{H}$ and $i \in I^{t(h)}(h)$, define $t_i^{\text{No}}(h)$ to be the time at which i sees the latest reply of No no later than period $t(h)$. That is,

$$t_i^{\text{No}}(h) := \max\{\tau \in \mathbb{N}_0 \mid R_j^\tau(h) = \text{No for some } (j, \tau) \in N \times \{0, \dots, t(h) - 1\} \cup \{(i, t(h))\}\}.$$

Now, we define, for each history $h \in \mathcal{H}$ and player $i \in I^{t(h)}(h)$, player $j (\neq i)$'s *relevant proposal* $P_j^{i\text{-rel}}(h)$ for i at h by

$$P_j^{i\text{-rel}}(h) := \begin{cases} P_j^{t_j^{\text{sp}}(h)}(h) & \text{if } t_i^{\text{No}}(h) \leq t_j^{\text{sp}}(h^{t(h)-1}) \\ X & \text{otherwise} \end{cases}.$$

In words, for each player i who speaks at h , her opponent j 's relevant proposal for i at the history h is the most recent announcement by j after the most recent No in terms of player i 's observation at h .

Player i , who speaks at history h , is ok with an alternative x at h if $\{x\}$ is the intersection of proposals that she made and saw in the past after the latest No. Put differently, player i is ok with x at a history if the intersection of her proposal and all the relevant proposals for her becomes the singleton set $\{x\}$. Formally, player $i \in I^{t(h)}(h)$ is *ok* with $x \in X$ at history $h \in \mathcal{H}$ if $\{x\} = P_i^{t(h)}(h) \cap \left(\bigcap_{j \in N \setminus \{i\}} P_j^{i\text{-rel}}(h)\right)$. The right-hand side is an intersection of two sets. The first is player i 's proposal in the current period. The second set is the intersection of her opponents' latest proposals after i observes the latest No (i.e., the intersection of her opponents' relevant

proposals). The condition is saying that the intersection of these two sets is a singleton set $\{x\}$. Loosely speaking, player i , who speaks at a history, is ok with x at that history if the intersection of all players' latest proposals after the most recent No that i has made or observed is the singleton set $\{x\}$.

The reason that we view this definition as capturing the idea of “ok” is two-fold. First, the second term in the intersection is what the other players have left as possibilities after someone has expressed dissatisfaction by announcing No. Proposing the first term such that the intersection becomes $\{x\}$ means that x is the only possibility anyone can interpret as what i has left as a possibility. If, on the other hand, the intersection consists of multiple alternatives including x , it is unclear if i is satisfied with x or simply wants to wait and see the opponents' responses to determine her future responses by not specifying a single alternative but restricting the set of possibilities. Second, player i could have said No as her current response, by which she would have been able to make the entire intersection a non-singleton unless her proposal itself is a singleton set. In such a case, she would have been in the situation where she could safely be interpreted as being *not* ok with x , unless she herself actively specifies x as the only possibility (recall that, by assumption, for any alternative $y \in X \setminus \{x\}$, i is able to announce P_i such that $y \in P_i$).

The consensual rule terminates a negotiation with an alternative at a given history once everyone is ok with the same alternative when she speaks after the most recent response of No. That is,

$$\varphi^{\text{con}}(h) := \begin{cases} x & \text{if each } j \in N \text{ is ok with } x \text{ at } h^{t_j^{\text{sp}}(h)} \text{ and } t^{\text{No}}(h) \leq t_j^{\text{sp}}(h) \\ \text{Continue} & \text{otherwise} \end{cases},$$

where we let $t^{\text{No}}(h)$ be the latest time at which some player says No under $h \in \mathcal{H}$, i.e.,

$$t^{\text{No}}(h) := \max \left\{ t' \in \mathbb{N}_0 \mid R_j^{t'}(h) = \text{No for some } (j, t') \in N \times \{0, \dots, t(h)\} \right\}.$$

We acknowledge that there would be many other sensible termination rules. We discuss other possible termination rules in Section B.6. Except for Section B.6, we only consider the consensual termination rule in order to focus on comparison of proposer and specification rules. Our exercise shows the usefulness of the idea to conduct

comparative statics by varying proposer and specification rules for a fixed termination rule.

Given a component game, the three rules, i.e., proposer, specification, and termination rules, generate the set $H(\subseteq \mathcal{H}^*)$ of histories of the negotiation game Γ and the set $Z(\subseteq H)$ of terminal histories of Γ . That is, $Z = \{h \in H \mid h \sqsubset h' \text{ implies } h' \notin H\}$.¹² These three rules are defined independently from each other, in order to examine how a change in a certain rule affects the outcome of the negotiation.

Strategies. For each player $i \in N$, a (pure) strategy of player i is a plan of what to announce in each history at which she speaks. Thus, letting $H_i := \{h \in H \setminus Z \mid i \in \rho(h)\}$ be the set of non-terminal histories after which player i makes a proposal, a *pure strategy* of player i is a mapping $s_i : H_i \rightarrow \{\text{Yes}, \text{No}\} \times \mathcal{P}_i$.¹³ The set of player i 's strategies is denoted by S_i . The set of strategy profiles is denoted by $S := \times_{i \in N} S_i$. Each strategy profile $s = (s_i)_{i \in N}$ induces a terminal history $h \in Z$.

Outcomes and Payoffs. The *outcome* of the negotiation induced by a strategy profile $s \in S$ under a termination rule φ is defined as follows. If the history $h = h(s) \in Z$ induced by s has a finite length and $\varphi(h) = x$, then x is an outcome of the negotiation induced by s . If $h \in Z$ has an infinite length, the outcome of the negotiation induced by s is defined as the *disagreement outcome* associated with h .

Each player's (vNM) payoff function in the extensive-form game $u_i : Z \rightarrow \mathbb{R}$ is given by $u_i(h) := d_i$ for any terminal history with infinite length (i.e., when the outcome is the disagreement outcome associated with h) and $u_i(h) := u_i(x)$ for any terminal history h which corresponds with an agreed-upon alternative $x \in X$ (i.e., when $\varphi(h) = x$). Note that we assume that the payoff from the disagreement outcome is independent of histories. Note also that there is no discounting.¹⁴ By a slight abuse of notation, each player's payoff function $u_i : S \rightarrow \mathbb{R}$ is defined by $u_i(h(s))$.

Individual Rationality and Pareto Efficiency. We denote by U the *feasible* payoff set: $U := \{u(x) \in \mathbb{R}^n \mid x \in X\}$, where $u(x) := (u_i(x))_{i \in N}$. We say that a vector

¹²Notice that Z includes all histories in H that have infinite lengths.

¹³Appendix B.5 deals with the case of behavioral strategies. Throughout the paper, we drop the reference to "pure"ness of strategies unless there is room for confusion.

¹⁴We discuss in Section B.4 an extension of our model where each player discounts the future.

of payoffs $v \in U$ is weakly individually rational (IR) if $v \geq d$.¹⁵ A vector of payoffs $v \in U$ is weakly Pareto efficient if there is no $v' \in U \setminus \{v\}$ such that $v' > v$. We also say that an alternative $x \in X$ is weakly individually rational (resp. weakly Pareto efficient) if $u(x)$ is weakly individually rational (resp. weakly Pareto efficient). We denote by $\text{IR}(U, d) := \{v \in U \mid v \geq d\}$ the set of weakly individually rational payoffs. Also, we denote by $\text{WP}(U) := \{v \in U \mid v \text{ is weakly Pareto efficient}\}$ the set of weakly Pareto-efficient payoffs. We assume that $\text{IR}(U, d)$ is a non-empty compact subset of \mathbb{R}^n .¹⁶

We define the *IR-Pareto-meet* (of U) by $U^M(U, d) := \{v \in \text{IR}(U, d) \mid v_i \geq w_i \text{ for some } w \in \text{WP}(U) \text{ and } i \in N\}$. That is, $U^M(U, d)$ (often shorthanded by U^M) is the set of weakly individually rational payoff profiles which give each player payoffs no less than her worst Pareto-efficient payoff.¹⁷ We also call the set of alternatives whose payoff profiles lie in the IR-Pareto-meet, $X^M := \{x \in X \mid u(x) \in U^M(U, d)\}$, to be the *IR-Pareto-meet* (of X).

Our solution concept is subgame perfect equilibrium (henceforth “SPE”) of the negotiation game $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi \rangle$.¹⁸ Let X^{SPE} be the set of SPE outcomes in X .

2.1 Illustration of the Consensual Termination Rule

In order to illustrate the consensual termination rule, consider negotiation games where the set of alternatives is the set of action profiles in the following normal-form game: $\langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$, where $N = \{1, 2\}$ is the set of players, A_i is the set of player i 's actions, and $u_i : A \rightarrow \mathbb{R}$ is i 's payoff function with $X = A := A_1 \times A_2$.

We assume that for the case when specifiability is unlimited, each player can announce an action profile as her proposal. Under limited specifiability, we assume

¹⁵For any vectors $x = (x_i)_{i \in N}, y = (y_i)_{i \in N} \in \mathbb{R}^n$, $x \geq y$ if and only if $x_i \geq y_i$ for all $i \in N$; and $x > y$ if and only if $x_i > y_i$ for all $i \in N$.

¹⁶The set $\text{IR}(U, d)$ is clearly compact if a set X of alternatives is a finite set or if X is compact and the players' payoff functions are continuous. Appendix B.8 considers the case where $\text{IR}(U, d)$ is the empty set.

¹⁷Rabin (1994) calls the set of payoff profiles which are at least as high as players' worst Pareto-efficient Nash-equilibrium payoff profiles to be the Pareto meet. Notice that we do not require the “Nash” restriction, and we added the modifier “IR-.”

¹⁸Our negotiation games do not satisfy “continuity at infinity” (Fudenberg and Levine (1983)) due to lack of discounting, and hence the “one-stage deviation” principle cannot be applied in investigating subgame perfect equilibria. Hence, we demonstrate that a specific strategy profile s^* constitutes a SPE by showing that each s_i^* is a best response to s_{-i}^* in any subgame.

	Specifiability	Proposal	History h	Termination
1	Unlimited	Asynchronous	$(h', (\text{No}, a), (\text{Yes}, a))$	Terminal
2	Unlimited	Synchronous	$(h', ((\text{No}, a), (\text{No}, a)))$	Terminal
3	Unlimited	Synchronous	$(h', ((\text{No}, a), (\text{No}, a')), ((\text{Yes}, a'), (\text{Yes}, a)))$	Non-terminal
4	Limited	Asynchronous	$(h', (\cdot, a_i), (\text{Yes}, a_{-i}), (\text{Yes}, a_i))$	Terminal
5	Limited	Asynchronous	$(h', (\cdot, a_i), (\text{Yes}, a_{-i}), (\text{Yes}, a'_i))$	Non-terminal
6	Limited	Synchronous	$(h', ((\cdot, a_1), (\cdot, a_2)), ((\text{Yes}, a_1), (\text{Yes}, a_2)))$	Terminal
7	Limited	Synchronous	$(h', ((\cdot, a_1), (\cdot, a_2)), ((\text{No}, a_1), (\text{No}, a_2)))$	Non-terminal

Table 1: List of terminal/non-terminal histories. To simplify the notation, we drop reference to the set of speakers. Note also that we let $a \neq a'$ and $a_i \neq a'_i$.

that each player can only specify her own action, and cannot specify the opponent's action.¹⁹ We consider two proposer rules: the synchronous case where both players move at each period, and the asynchronous case where player 1 moves at odd periods and player 2 moves at even periods.

Table 1 shows, for each pair of specification and proposer rules, whether each history h is terminal or not under the consensual rule. Rows 3 and 5 are especially worth explaining. For Row 3, h is not terminal because two players are ok with different action profiles a and a' , and in such a circumstance they would need more conversations to reach a consensus.²⁰ For Row 5, h is not terminal because, despite the fact that both players' latest responses are Yes, player $-i$ is ok with (a_i, a_{-i}) in period $t - 1$ while player i is ok with (a'_i, a_{-i}) in period t .

3 Benchmark Cases

We proceed with two benchmark observations. The first is that any alternative which is not weakly individually rational cannot be sustained as a SPE outcome under the consensual termination rule, for any proposer and specification rules. The reason is that, if there existed a SPE outcome that makes some player worse than the disagree-

¹⁹Formally, the case of unlimited specifiability corresponds to $\mathcal{P}_i = \{\{a\} \mid a \in A\}$, while that of limited specifiability corresponds to $\mathcal{P}_i = \{\{a_i\} \times A_{-i} \mid a_i \in A_i\}$. Hereafter, we abuse notation to denote by a the proposal $\{a\}$ under unlimited specifiability and by a_i the proposal $\{a_i\} \times A_{-i}$ under limited specifiability.

²⁰As an example, consider a couple exchanging emails about their plan for the next day. Suppose that after a woman expresses her willingness to go to ballet and a man expresses his willingness to go to a soccer match, they simultaneously send replies to each other in which the woman writes "Yes, let's go to soccer" and the man writes "Yes, let's go to ballet." Unless there is some predetermined rule, their email exchanges would need to continue to settle on a single plan for the day.

ment outcome, then such a player would be able to profitably deviate by announcing (No, P_i) such that P_i includes a weakly individually rational alternative at every history at which it is her turn to speak.²¹

Proposition 1. *For any negotiation $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$, any SPE outcome of Γ is weakly individually rational.*

The second observation is that, we obtain a “folk theorem” under synchronous proposer rules: any weakly individually rational alternative can be supported as a SPE outcome under the consensual termination rule, irrespective of specification rules.²²

Proposition 2. *For any negotiation $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$ such that ρ is synchronous, every $x \in X$ with $u(x) \geq d$ is a SPE outcome.*

To see this, fix x and a profile of proposals $(P_i)_{i \in N}$ such that $\{x\} = \bigcap_{i \in N} P_i$. The following is a SPE which supports x as its outcome. Each player i announces (Yes, P_i) if the announcement profile in the last period at which players have moved entails no deviation, while they announce (No, P_i) otherwise.²³ No player has an incentive to deviate because, given the opponents’ strategies, it is only x or d that can be an outcome after any history.

We can sustain such strategy profiles in SPE because no player can influence the opponents’ future actions by committing to a proposal, just as in repeated coordination games where we are unable to rule out a repetition of an inefficient Nash equilibrium. Such lack of commitment is partially overcome when a proposer rule is asynchronous.

Propositions 1 and 2 imply the following corollary:

Corollary 1. *For any negotiation $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$ such that ρ is synchronous, $x \in X$ is a SPE outcome if and only if $u(x) \geq d$.*

²¹Such P_i exists by assumption. When player i announces (No, P_i), she is not ok with any alternative when P_i is not a singleton and she is ok with y if $P_i = \{y\}$. Thus, any player i can guarantee herself at least the disagreement payoff by keeping announcing (No, P_i).

²²Kalai, Kalai, and Samet (2010) show a “folk theorem” with synchronous moves in the context of what they call the (two-player) “commitment games.”

²³In particular, players announce (Yes, P_i) at the first period at which they move.

4 Specificifiability-Free Results with Asynchronous Moves

The previous section shows lack of prediction under synchronous-move negotiations. In the rest of the paper, we focus on asynchronous proposer rules to see how such a conclusion changes. It will turn out that the way in which asynchronicity helps narrow down our prediction depends on the structures of component games and specification rules. First, this section provides predictions free from specification rules. Section 5 then discusses how the limitation on specification rules change the predictions.

4.1 Negotiations with a Common-Interest Alternative

We say that a negotiation $\langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$ has a *common-interest alternative* $x^* \in X$ if $u_i(x^*) > u_i(x)$ for all $x \in \text{IR}(X, d) \setminus \{x^*\}$ for all $i \in N$.²⁴

Theorem 1. *Any negotiation $\langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$ such that ρ is asynchronous has a unique SPE outcome x^* if and only if it has a common-interest alternative x^* .*

The theorem applies to general n -player cases. To see what this theorem implies, consider the following two examples in the context of two-player cases.

Example 1. The component game is given by $X = A_1 \times A_2$, where A_i and u_i are those of the “tacit coordination game” studied by Bryant (1983) and Van Huyck, Battalio, and Beil (1990): each player chooses an effort level $a_i \in A_i := [0, 1]$, and her payoff is $u_i(a_1, a_2) = 2 \min\{a_1, a_2\} - \frac{1}{2}a_i$. We let $d = (0, 0)$. The feasible payoff set and the unique SPE payoff profile are depicted in the left panel of Figure 2. The normal-form game has a continuum of Nash equilibria $\{(a_1, a_2) \mid a_1 = a_2 \in [0, 1]\}$, among which the action profile $(1, 1)$ Pareto-dominates all other Nash equilibria. Our result shows that the latter is the *unique* SPE outcome. \square

Example 2. The component game is given by $X = A_1 \times A_2$, where $A_i = \{C_i, D_i\}$ and u_i are those of the Prisoners’ Dilemma game depicted in Figure 2, where we set a disagreement payoff $d_i \in (-2, 0]$ for each $i \in N$. The normal-form game has a unique Nash equilibrium (D_1, D_2) . The Pareto frontier consists of three points, corresponding to action profiles (C_1, C_2) , (C_1, D_2) , and (D_1, C_2) . However, (C_1, C_2) is a unique weakly individually rational and Pareto-efficient action profile, and thus the

²⁴Note that, by assumption, $x^* \in \text{IR}(X, d)$ holds because $\text{IR}(X, d)$ is nonempty.

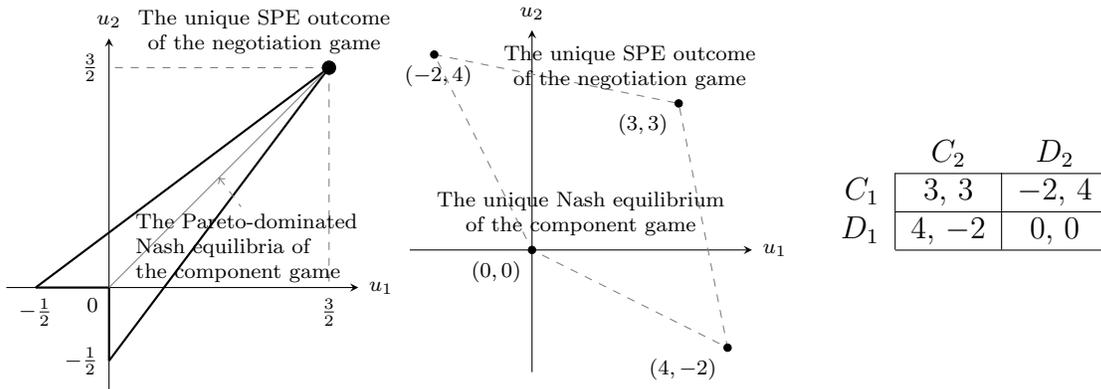


Figure 2: The feasible, Nash, and SPE payoff sets for the tacit coordination game (left). A Prisoners' Dilemma game: the payoff matrix (right) and the feasible, Nash, and the SPE payoff sets with $d_i \in (-2, 0]$ for each player i (center).

theorem applies. Our result shows that (C_1, C_2) is a *unique* SPE outcome.²⁵ Thus, the set of Nash equilibria of the component game and the set of SPE outcomes in the negotiation game can be disjoint. \square

Existence of an equilibrium will be shown in the next subsection (Corollary 2) in the most general environment.²⁶ Here we explain the intuition behind why only common-interest alternative can be an equilibrium outcome, using the Prisoners' Dilemma game in Example 2. Consider the case with limited specifiability in which each player can only announce their action (a similar argument applies to the case with any specifiability condition). First, notice that the only individually rational payoff profiles are $(0, 0)$ and $(3, 3)$, so by Proposition 1, the two players' payoffs have to be equal in any SPE.

Now, after any history of the form $h = (h^{t(h)-2}, (\text{Yes}, C_1), (\text{Yes}, C_2))$, player 1 can terminate the negotiation with (C_1, C_2) by announcing (Yes, C_1) , so that 1's payoff must be no less than 3 after history h , which is her best possible equilibrium payoff conditional on h^t . Thus, (C_1, C_2) is the unique outcome of the subgame starting after h in any SPE.

Hence, given any history of the form $h^{t(h)-1} = (h^{t(h)-2}, (\text{Yes}, C_1))$, player 2 an-

²⁵Kalai (1981) studies a pre-play communication model in which the Prisoners' Dilemma game is the underlying game, and shows that (C_1, C_2) is a unique equilibrium outcome.

²⁶The proof for existence is constructive. For example, in Example 1, one can construct a SPE strategy profile in which each player announces $(\text{Yes}, 1)$ as long as the opponent has announced $(\cdot, 1)$ in the previous period; otherwise, she announces $(\text{No}, 1)$. The actual construction in the proof is more contrived in order to deal with general component games and specifiability conditions.

nouncing (Yes, C_2) guarantees his opponent 1 a payoff of 3, which in turn guarantees himself a payoff of 3. In other words, (C_1, C_2) is the unique outcome of the subgame starting after $h^{t(h)-1}$ in any SPE.

Finally, at the start of the negotiation, player 1's announcement (Yes, C_1) induces the history $(h^0, (\text{Yes}, C_1))$, which is of the form $h^{t(h)-1}$ as above. Hence player 1 can guarantee herself a payoff of 3, which means player 2's minimum SPE payoff is 3 as well. Thus, in any SPE, the payoff must be 3 for each player $i \in N$. This implies that the unique SPE outcome is the unique individually rational Pareto-efficient action profile (C_1, C_2) .

In the above argument, we used the fact that two players must receive equal payoffs in any SPE. The argument actually depends only on the fact that there is a common action profile that strictly dominates all other profiles, and it is why the result can be extended to any negotiation with a common-interest alternative as in Theorem 1. In other words, the theorem depends on the fact that if i receives the best individually rational payoff under a given strategy profile, it fully pins down $-i$'s payoff under that strategy profile. The next subsection deals with the case in which there is no common interest alternative, and shows that multiple negotiation outcomes exist.

In standard repeated Prisoners' Dilemma games, each player i unconditionally choosing D_i is a SPE, and one may wonder why such a strategy profile cannot constitute a SPE in our negotiation game. The reason is that a termination of a negotiation is endogenously determined in an equilibrium, and it is optimal for player 1 to respond with C_1 given history h as above. Then one may ask again: Why does player 1 not want to terminate with D_1 ? The reason is that, under history h , announcing C_1 and D_1 have asymmetric implications on the process of negotiation: C_1 leads to a termination while D_1 does not. Proposing D_1 cannot terminate the negotiation because player 2 would not be ok with an action profile (D_1, C_2) .

4.2 Negotiations without a Common-Interest Alternative

We now turn to negotiations that do not have a common interest alternative and seek for predictions free from specification rules.

Theorem 2. *For any negotiation $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$, every $x \in X^M$ is a SPE outcome.*

If the (individually rational) Pareto frontier consists of multiple points, then the fact that player i receives the best individually rational payoff does not pin down $-i$'s payoff. Thus the commitment power is weaker, leading to a possibility of punishment. As a consequence, the set of equilibrium payoffs consists of multiple points. At an extreme, if the given normal-form game is “strictly competitive” (that is, $u_1(a) \geq u_1(a')$ if and only if $u_2(a) \leq u_2(a')$ for all $a, a' \in A$), Theorem 2 implies that any individually rational payoff profile can be sustained as a SPE outcome.

The proof consists of two steps. First, we show that for each player i , her worst individually-rational alternative on the Pareto frontier, $x^{(i)}$, can be sustained in a SPE under the consensual rule. Second, we show that any points in the IR-Pareto-meet can be attained by using $x^{(i)}$ to punish i 's deviations.²⁷ We note that showing the first step is by no means trivial, as we need to make sure that there exists a strategy profile in which players take best responses even off the equilibrium path: Off the equilibrium path, some players may have already been ok with an alternative x , and it may be of the remaining players' best interest to agree on x even if x is not a SPE outcome. Checking if such an agreement is of “best interest” of these remaining players is complicated because each player off the path needs to correctly forecast the future actions by the opponents.

Under our assumption that $\text{IR}(U, d)$ is a non-empty compact set, the IR-Pareto-meet is always non-empty. Hence the following holds.

Corollary 2. *Any negotiation $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$ has a SPE.*

Theorem 2 shows that the set of SPE alternatives always contains the IR-Pareto meet, X^M . Next we examine how large the SPE set can be. Hereafter, we restrict attention to two-player negotiations for which sharp results are obtained.

First, we show that it is without loss of generality to restrict attention to the proposer rule in which player 1 moves at odd periods and player 2 moves at even periods. This lemma is going to be convenient in narrowing down the possible histories that can arise under given strategy profiles.

²⁷Such a strategy profile corresponds to a situation where there is a preset focal alternative that both parties know is Pareto-inefficient, and they believe that proposing something unexpected (things that are not along the line with the focal alternative) would provoke the opponent's antipathy and makes him aggressive in the future negotiation (of course, to the extent that such aggressiveness is supported under best response conditions).

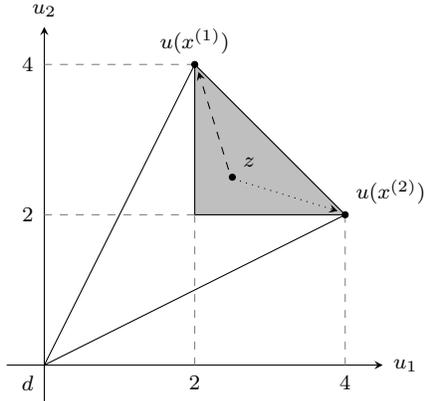


Figure 3: Illustration of the proof of Theorem 2: The shaded area is $\{u(x) \mid x \in X^M\}$. The dashed and dotted arrows indicate punishment for players 1 and 2, respectively.

Lemma 1. *Fix a two-player negotiation game $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$ such that ρ is asynchronous. Then, $v \in U$ is a SPE payoff profile of Γ if and only if it is a SPE payoff profile of $\Gamma^* = \langle G, d, \rho^{\text{asyn}}, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$, where the proposer rule ρ^{asyn} lets player 1 propose in odd periods and player 2 propose in even periods.*

The intuition is that under the consensual termination rule, only the latest announcement by each player matters. Thus even if a player can make proposals for consecutive periods, only the last proposal can ever matter. The implication of this result is that, under the asynchronous rule, the frequency with which a player can speak does not matter for the set of SPE payoffs.²⁸

Now, define:

$$\begin{aligned}
 v^{[i,M]} &:= \max\{v_i \mid v \in \text{IR}(U, d)\}; \\
 v^{[i,m]} &:= \min\{v_i \mid v \in \text{IR}(U, d) \text{ and } v_{-i} \geq v^{[-i,M]}\}; \text{ and} \\
 \underline{v}_i &:= \min\{v_i \mid v \in \text{IR}(U, d) \text{ and } v_{-i} \geq v^{[-i,m]}\}.
 \end{aligned}$$

²⁸This is in contrast to the case with discounting, where the frequency determines the payoffs. We will be detailed on the comparison with the case with discounting in Section B.4. The result also implies that there is no first- or second-mover advantage. Intuitively, this is because each player has to be ok with an alternative for it to be an outcome under the consensual termination rule. Under Bhaskar's (1989) termination rule that we describe as a special case of our model in Appendix B.9, on the other hand, there may exist first- or second-mover advantages. For example, if the component game is that of Battle of the Sexes, there is a unique SPE outcome in which the second-mover receives the best feasible payoff.

For each $i \in N$, we call \underline{u}_i the *worst Pareto-guaranteeing payoff* for player i . To make the dependence of $\underline{u} := (\underline{u}_1, \underline{u}_2)$ on (U, d) clear, we sometimes denote it by $\underline{u}(U, d)$.

Proposition 3 (The worst Pareto-guaranteeing payoffs are lower bounds). *Fix a two-player negotiation $\Gamma = \langle G, d, \rho, (P_i)_{i \in N}, \varphi^{\text{con}} \rangle$ such that ρ is asynchronous. If an alternative $x \in X$ is sustained as a SPE outcome of Γ , then $u(x) \geq \underline{u}$.*

Let us explain the intuition for this proposition. Consider a history at which i has announced P_i such that there exists P_{-i} such that $P_i \cap P_{-i} = \{x\}$ with $u_i(x) = v^{[i, M]}$. Since $v^{[i, M]}$ is the maximum payoff that i can ever receive in a SPE, the definition of $v^{[-i, m]}$ implies that player $-i$ can guarantee herself the payoff $v^{[-i, m]}$ at that history. Observe that $v^{[-i, m]}$ is player $-i$'s minimum weakly individually rational and Pareto efficient payoff.

Solving backwards, in a similar vein, at any history, each player i can guarantee herself the minimum payoff \underline{u}_i such that her opponent $-i$ can obtain $-i$'s minimum weakly individually rational and Pareto efficient payoff. Hence, \underline{u}_i is a lower bound of player i 's utility in the negotiation game is the worst Pareto-guaranteeing payoff. In words, each player i can guarantee herself the minimum payoff from the set of alternatives which yield player $-i$ at least as high as player $-i$'s minimum weakly individually rational and weakly Pareto-efficient payoff.

Recall that folk theorem holds under synchronous proposer rules (Proposition 2). The implication of Proposition 3 is that certain payoffs may not be achievable in SPE under asynchronous proposer rules. That is, asynchronicity helps narrow down the set of SPE payoffs.

The next example illustrates the computation of the worst Pareto-guaranteeing payoffs.

Example 3. Let $U = \text{conv}(\{(0, 0), (4, 2), (2, 4)\})$. In the left panel of Figure 4, we set $d = (0, 0)$. By inspection, we obtain $(v^{[1, M]}, v^{[2, m]}) = (4, 2)$, $(v^{[1, m]}, v^{[2, M]}) = (2, 4)$, and $(\underline{u}_1, \underline{u}_2) = (1, 1)$.

In the middle panel of Figure 4, we let $d = (\frac{3}{2}, 3)$. We have $(v^{[1, M]}, v^{[2, m]}) = (3, 3)$, $(v^{[1, m]}, v^{[2, M]}) = (2, 4)$, and $(\underline{u}_1, \underline{u}_2) = d$.

Suppose, on the other hand, that $U = \{(0, 0), (4, 2), (2, 4)\}$ as in the right panel of Figure 4. Let $d = 0$. We have $(v^{[1, M]}, v^{[2, m]}) = (4, 2)$, $(v^{[1, m]}, v^{[2, M]}) = (2, 4)$, and $(\underline{u}_1, \underline{u}_2) = (2, 2)$. □

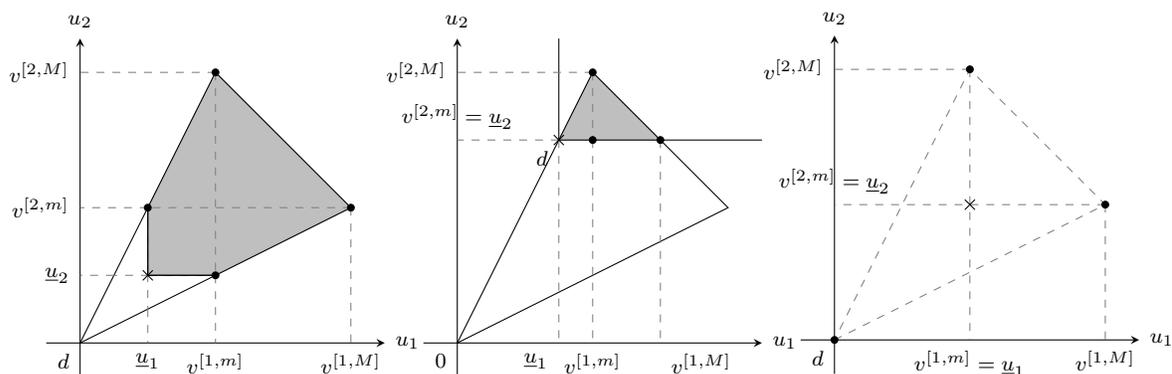


Figure 4: The computation of the worst Pareto-guaranteeing payoffs: the case with $d = 0$ (left); the case with $d = (\frac{3}{2}, 3)$ (middle); and the case with finite X and $d = 0$ (right).

5 Unlimited vs. Limited Specifiability with Asynchronous Moves

5.1 Unlimited Specifiability

We first analyze negotiations under unlimited specifiability. The main result of this section is that the set of SPE payoffs can be completely characterized as the IR-Pareto-meet under unlimited specifiability. To show this result, we first determine player i 's lower bound of SPE payoffs when i 's specifiability is unlimited.

Proposition 4 (One player has unlimited specifiability). *Fix a two-player negotiation game $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$ where \mathcal{P}_i is unlimited, If x is a SPE outcome, then $u_i(x) \geq v^{[i,m]}$.*

Roughly, this result follows because, if a player's specifiability is unlimited, she can commit to choosing her opponent's best weakly individually rational alternative. Formally, suppose that player i 's specifiability is unlimited. Let $x^{(i)}$ be the alternative that gives player $-i$ the best payoff in $\text{IR}(X, d)$ and gives player i her least weakly Pareto-efficient and weakly individually rational payoff.²⁹ Note that $\{x^{(i)}\} \in \mathcal{P}_i$ for player i because her specifiability is unlimited. Given any non-terminal history at which i speaks, if she announces (No, $\{x^{(i)}\}$), then (i) player $-i$ can guarantee himself

²⁹The existence of such an alternative follows from compactness of $\text{IR}(X, d)$ and the assumption that $n = 2$, and is proven in the Appendix.

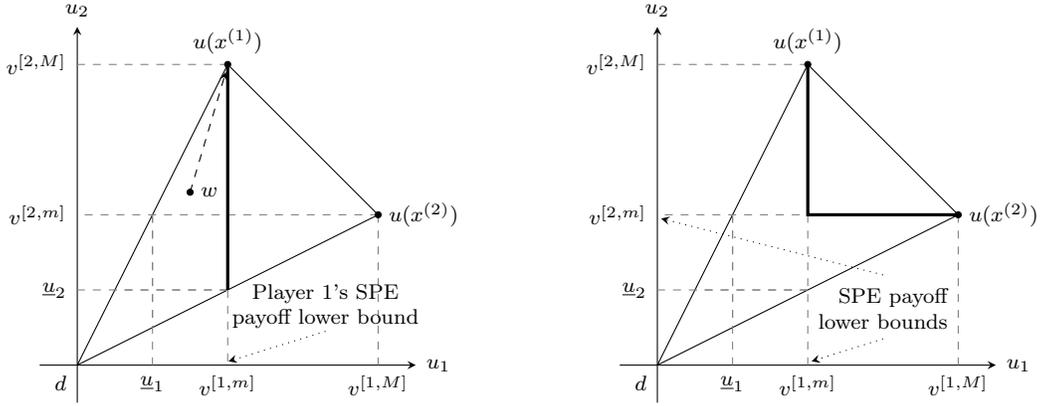


Figure 5: The left panel illustrates the proof of Proposition 4 with $i = 1$: The dashed arrow indicates player 1's deviation to $x^{(1)}$. The lower bound of player 1's SPE payoff is given by $v^{[1,m]}$. The right panel illustrates Corollary 3 (specifiability of each player is unlimited). In Corollary 3, $X^{\text{SPE}} = X^M$.

a payoff no less than $u_{-i}(x^{(i)})$ because he can terminate the negotiation by announcing (Yes, P_{-i}) such that $x^{(i)} \in P_{-i}$, and (ii) player i can guarantee herself an individually rational payoff because she can keep announcing (No, $\{x^{(i)}\}$). These two properties imply that, after any non-terminal history at which i speaks, she can guarantee herself her least weakly Pareto-efficient and weakly individually rational payoff. Hence, in any SPE, player i can guarantee herself her least weakly Pareto-efficient and weakly individually rational payoff. This completes the proof.

Combining this result with that of Theorem 2, we obtain the following corollary.

Corollary 3. *Fix a two-player negotiation $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$ such that ρ is asynchronous and $(\mathcal{P}_i)_{i \in N}$ is unlimited. Then an alternative $x \in X$ is supported as a SPE outcome if and only if it is in the IR-Pareto-meet (i.e., $x \in X^M$).*

5.2 Limited Specifiability

In this section, we consider the case of limited specifiability. In Section 5.2.1, we first define the notion of unilaterally improvability and, with that notion, give a series of results on SPE outcomes. Then we provide an example in which whether a particular alternative can be supported as a SPE outcome depends on the specification rule. In Section 5.2.2, we further investigate this dependence, showing that the set of SPE outcomes can vary from X^M to the set of all alternatives x with payoffs $u(x) \geq \underline{u}$.

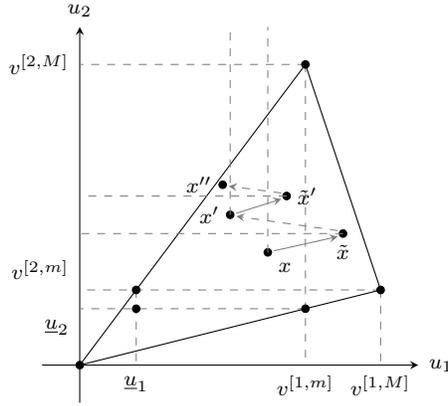


Figure 6: Intuition for the proof of Theorem 3. The solid arrows indicate deviations and the dashed arrows indicate punishments.

5.2.1 Unilateral Improvability and SPE Payoffs

In the proof of Proposition 4, it was important that once player i announces the alternative that induces $-i$'s best payoff, $-i$ can terminate the negotiation. This is not the case with limited specifiability, as i 's announcement cannot pin down the exact outcome. This means that reaching a consensus upon a potential deviation needs longer periods, so there is a greater scope for punishments to such a deviation. This leads to a larger set of SPE payoff profiles under limited specifiability.

Whether the increased length until the termination supports an alternative as a SPE outcome depends on the detail of the way in which specifiability is limited and the payoff structure. The following joint condition on specifiability and payoff structure is crucial in characterizing the SPE payoff set under limited specifiability: We say that $x \in X$ is *unilaterally improvable for player i* if for all (P_1, P_2) such that $P_1 \cap P_2 = \{x\}$, there exists (P'_i, x') $\in \mathcal{P}_i \times X$ such that $P'_i \cap P_{-i} = \{x'\}$ and $u(x') > u(x)$. That is, player i can unilaterally deviate and create an intersection that Pareto-improves upon x .

Theorem 3. *Fix a two-player negotiation $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$ such that X is finite, ρ is asynchronous and $(\mathcal{P}_i)_{i \in N}$ is limited. Then, $X^{\text{SPE}} = X^M$ if and only if, for each i , every $x \in X$ such that $u_i(x) < v^{[i,m]}$ and $u_{-i}(x) \geq v^{[-i,m]}$ is unilaterally improvable for i .*

The proof of the “If” part consists of two steps. First we prove that, under

the given condition, no alternative x such that $u_i(x) < v^{[i,m]}$ and $u_{-i}(x) \geq v^{[-i,m]}$ is a SPE outcome. Second, we use this result to show that no alternative x with $u(x) < (v^{[1,m]}, v^{[2,m]})$ and $u(x) \geq \underline{u}$ is a SPE outcome.

The idea of the first step uses the payoff bounds given by unilateral improvability and equilibrium conditions. We explain the intuition using Figure 6. Suppose that every alternative outside X^M is unilaterally improvable, and an alternative x with $u_1(x) < v^{[1,m]}$ and $u(x) \geq \underline{u}$ is a SPE outcome. Then unilateral improvability implies that player 1 has a deviation to some \tilde{x} that guarantees player 2 a payoff strictly greater than $u_2(x)$. By the equilibrium condition, such a deviation has to be punished by an off-equilibrium outcome x' that gives player 1 a payoff no more than $u_1(x)$. Hence $u_1(x') \leq u_1(x)$ and $u_2(x') > u_2(x)$. Now, since player 1 has to become ok with such x' at such an off-path history (this part needs a bit more elaboration that we make explicit in the proof), and thus there exists another alternative \tilde{x}' with $u_1(\tilde{x}') \leq u_1(x')$ and $u_2(\tilde{x}') > u_2(x')$ that both players can be ok at an off-path history. Again, such a deviation has to be punished by an off-equilibrium outcome x'' that gives player 1 a payoff no more than $u_1(x')$. Going forward, we need to be able to find an infinite sequence of alternatives that goes to the north-west direction given by alternations of deviations and punishments, but this contradicts the assumption that X is finite. Indeed, the Appendix provides a counterexample for the case with infinite X .

For the second step, roughly, we consider player 1's deviation to announce a proposal that includes an alternative with payoff $(v^{[1,M]}, v^{[2,m]})$. Such a deviation has to be punished by a continuation strategy that leads to an outcome giving player 1 a payoff less than $v^{[1,m]}$ and player 2 a payoff greater than $v^{[2,m]}$. But by assumption such an alternative is unilaterally improvable, and by a similar argument as for the first step, such an alternative cannot be supported in the continuation play.

The “only if” part is a consequence of the following stronger result:

Proposition 5. *Fix a two-player negotiation $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$, ρ is asynchronous and $(\mathcal{P}_i)_{i \in N}$ is limited. Then, every $x \in X$ with the following three properties is a SPE outcome: (i) $u(x) \geq \underline{u}$; (ii) x is not unilaterally improvable for any player i with $u_i(x) < v^{[i,m]}$; and (iii) there exists $(y^1, y^2) \in X^2$ such that, for each $i \in \{1, 2\}$, $u_i(y^i) \leq u_i(x)$ and $u_{-i}(y^i) \geq v^{[-i,m]}$ and y^i is not unilaterally improvable for player i .*

The proof of Proposition 5 is constructive. For any x with the property stated in the proposition, we construct a SPE strategy profile that induces x as an outcome.

	L	R
U	0, 0	2, 4
D	4, 2	1, 3

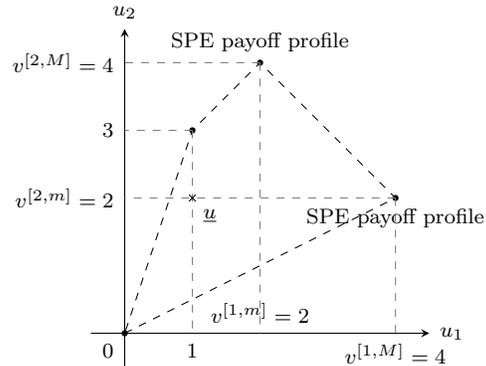


Figure 7: An example where the worst Pareto-guaranteeing payoff is a loose lower bound: the payoff matrix (left) and the feasible and SPE payoff sets (right).

Specifically, we sustain x by a threat to punish player i by an off-path outcome y^i . The detailed description of the intuition for such a strategy profile is explained in the context of the special case treated by Corollary 5. A new complication under limited specifiability is that we need to analyze more cases than under unlimited specifiability because it takes longer periods to reach an agreement under the consensual termination rule.

For a class of negotiations generated by normal-form games, the following corollary to Theorem 2 and Proposition 5 shows that Nash equilibria in the given normal-form game are always the SPE outcomes of the corresponding negotiation.

Corollary 4. *Consider the negotiation defined by a two-player normal-form game in Section 2.1. If specifiability is limited, then any Nash equilibrium $a \in A$ with $u(a) \geq \underline{u}$ is not unilaterally improvable, and hence is a SPE outcome.*

These results show that it is possible for negotiations under limited specifiability to lead to more SPE outcomes, but the extent to which this happens depends on the given problem.

Example 4. Consider the component game in the left panel of Figure 7 and a negotiation game with asynchronous proposer rule and limited specifiability (i.e., each player can only announce their action). Figure 7 depicts the computation of \underline{u} in this case, and in particular a payoff profile (1,3) is at least as high as the worst Pareto-guaranteeing payoff profile $\underline{u} = (1, 2)$.

Note that (D, R) is the only alternative outside X^M and is above $\underline{u} = (1, 2)$. Also, it is unilateral improvable because in that definition, we can let $P_1 = \{D\} \times \{L, R\}$,

$P_2 = \{U, D\} \times \{R\}$, and $P'_1 = \{U\} \times \{L, R\}$.

As a consequence, the payoff profile $(1, 3)$ *cannot* be sustained in a SPE. To see this, suppose that it is sustained under some SPE. In order to sustain this payoff profile, player 2 must announce (\cdot, R) at some point on the equilibrium path at which the negotiation does not terminate. But then, player 1 can announce (Yes, U) , after which player 2 has an option to say (Yes, R) that ends the negotiation. This means that, after player 2's announcement of (\cdot, R) that does not end the negotiation which would be a necessary step for $(1, 3)$ to be sustained, player 1 can guarantee a payoff of 2. This means that $(1, 3)$ cannot be sustained in a SPE.

Note that if we replace the payoff profile under (U, L) with $(1, 3)$ and the one under (D, R) with $(0, 0)$, then $(1, 3)$ is sustainable under SPE. This is because if player 1 deviates by proposing D , then we can have 2 announce (No, L) on the equilibrium path. If player 2 deviates by not announcing (No, L) , players switch to the Pareto-efficient outcome (D, L) (with payoffs $(4, 2)$). Formally, (U, L) is not unilaterally improvable.

The reason for the difference is as follows: Under the game in Figure 7, an alternative that (i) strictly Pareto-dominates $(1, 3)$ and (ii) gives player 2 the best feasible payoff is included in player 2's imprecise proposal that would sustain the given payoff profile. On the other hand, no such alternative can be found under the modified game in the proposal sustaining the given payoff profile (player 1's proposal U contains a strictly Pareto-dominating alternative with a payoff profile $(2, 4)$, but player 2's payoff (which is 3) is higher than his worst Pareto-efficient payoff (which is 2)).

The construction of the component game in the proof of Part 1 of Corollary 5 intends to avoid this problem. \square

Theorem 3 and Proposition 5 leave it open whether an alternative in $X \setminus X^M$ that is unilaterally improvable for i can be supported as a SPE outcome when there exists another alternative in $X \setminus X^M$ that is not unilaterally improvable for i . The Appendix B.2 provides an example in which, $x \in X \setminus X^M$ with $u(x) \geq \underline{u}$ is unilaterally improvable but it is a SPE outcome when there is $y \in X \setminus X^M$ with $u(y) \geq \underline{u}$ that is not unilaterally improvable. This can happen because the aforementioned sequence of deviations and punishment can terminate at such y .

5.2.2 Tightness of Payoff Bounds

Below we show that for any non-empty compact payoff set \bar{U} , there exists a negotiation game whose feasible payoff set is \bar{U} in which bounds are tight. We also show that for any non-empty compact payoff set \bar{U} , there exists a negotiation game whose feasible payoff set is \bar{U} in which (i) the lower bounds do not coincide with the worst Pareto-efficient and individually rational payoff profile and (ii) the SPE payoff set is the IR-Pareto-meet.

Corollary 5 (Existence of games where the bounds are tight). *Fix a set $\bar{U} \subseteq \mathbb{R}^2$, a vector $d \in \mathbb{R}^2$ such that $\{v \in \bar{U} \mid v \geq d\}$ is a non-empty compact set, and an asynchronous proposer rule ρ . There exists a pair of two-player negotiations (Γ^L, Γ^H) where for each $k \in \{L, H\}$, $\Gamma^k = \langle G^k, d, \rho, (\mathcal{P}_i^k)_{i \in N}, \varphi^{\text{con}} \rangle$ with $G^k = \langle N, X^k, (u_i^k)_{i \in N} \rangle$, $\bar{U} = \{u^k(x) \in \mathbb{R}^2 \mid x \in X^k\}$, $(\mathcal{P}_i^k)_{i \in N}$ is limited, and the following are true:*

1. *An alternative $x \in X$ is sustained as a SPE outcome of Γ^L if and only if $u^L(x) \geq \underline{u}(\bar{U}, d)$.*
2. *An alternative $x \in X$ is sustained as a SPE outcome of Γ^H if and only if $u(x) \in U^M(\bar{U}, d)$.*

Corollary 5 shows a certain tightness of the worst Pareto-guaranteeing payoffs as lower bounds of SPE payoffs. In order to show this result, we first construct a component game that is a normal-form game in which each player's action corresponds to her own payoff. The payoff profile from an action profile in this example is the same as the action profile if it is feasible (i.e., it is in the set of payoffs \bar{U}), and otherwise we set it to be a sufficiently low feasible payoff profile. With this specification, no alternative is unilaterally improvable.

Let us explain how to achieve each payoff profile v with $v \geq \underline{u}$ in SPE under limited specifiability, using Figure 8. We have already explained that points on the Pareto frontier as well as point z is achievable under any specifiability condition (arrows from z indicates punishment).

We first explain why w is achievable. Consider an on-path strategy analogous to the above one. The following is a punishment strategy that can be used to sustain w . Suppose first that player 2 deviates by announcing v'_2 , which constitutes part of (v'_1, v'_2) which Pareto-dominates w . Then, from that subgame on, players switch

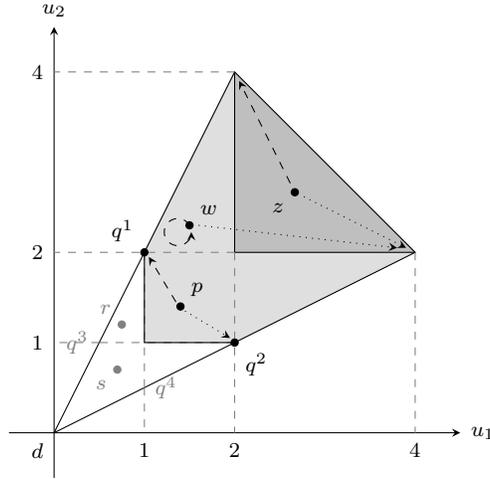


Figure 8: Description of SPE strategies: The dashed arrow indicates punishment for player 1, and the dotted arrow does for player 2.

to another SPE that supports $(4, 2)$. The dotted arrow in the figure shows such a punishment for player 2. Suppose next that player 1 deviates by announcing an action v'_1 , which again constitutes part of (v'_1, v'_2) Pareto-dominates w . If she does this, then players do not switch but continue playing w . The dashed arrow in the figure shows such a punishment for player 1. To do this, player 2 responds with (No, w_2) to 1's deviation.

Recall that, under unlimited specifiability, point w is not a SPE payoff profile. In particular, a deviation to announcing (Yes, v') is profitable. The reason that player 2 does not have an incentive to announce, say, (Yes, v'_2) under limited specifiability is that if he does so, then players switch to a SPE which supports $(4, 2)$ and it gives a lower payoff to player 2 than w . Here, it is important that player 2 cannot terminate the negotiation by himself. We used longer periods necessary to terminate a negotiation under limited specifiability to construct a SPE as above, exploiting the wider scope for punishment.

Analogously, any payoff profile which is at least as good as q^1 for player 2 can be sustained in a SPE under limited specifiability. Similarly, any payoff profile which is at least as good as q^2 for player 1 can be sustained.

Finally, the payoff profile p can be sustained under limited specifiability by using q^1 and q^2 as punishments. These arguments show that the set of SPE payoffs under unlimited specifiability is as depicted in the middle panel of Figure 1, while that under

	L	R
U	1, 3	2, 4
D	4, 2	0, 0

Table 2: Payoff matrix for Example 5

limited specifiability is as depicted in the right panel of Figure 1.

One may wonder why payoffs such as r and s cannot be sustained, by perhaps using q^3 and q^4 as punishments. The reason is that the sequence of potential punishments that such a punishment would induce would be too long that the negotiation could terminate in the meantime. This is parallel to the intuition that, under unlimited specifiability with which the length until the termination of a negotiation is even shorter, w and p cannot be sustained.

5.2.3 Comparative Statics with respect to Specifiability

For any given $\mathcal{P}_i \subseteq 2^X$, let $\mathcal{F}(\mathcal{P}_i) = \{P'_i \in 2^X \mid \text{there is } P_i \in \mathcal{P}_i \setminus \{\emptyset\} \text{ such that } P_i \subseteq P'_i\}$. We say that \mathcal{P}_i is more limited than \mathcal{P}'_i if $\mathcal{F}(\mathcal{P}_i) \subseteq \mathcal{F}(\mathcal{P}'_i)$. Note that the “more limited” relation is a reflexive and transitive binary relation on 2^{2^X} , and any \mathcal{P}_i that is unlimited is a minimal element in such binary relations.

Proposition 6. *Suppose that $x \in X$ is a SPE outcome of a two-player negotiation $\langle G, d, \rho, (\mathcal{P}_i, \mathcal{P}_{-i}), \varphi^{\text{con}} \rangle$. If \mathcal{P}'_i is more limited than \mathcal{P}_i for player i , then x is a SPE outcome of the negotiation $\langle G, d, \rho, (\mathcal{P}'_i, \mathcal{P}_{-i}), \varphi^{\text{con}} \rangle$.*

One may wonder if a player’s specifiability becomes more limited then that has a favorable effect on the opponent’s payoff. This is not necessarily true. Consider the following example.

Example 5. Consider the payoff matrix in Table 2. Let \mathcal{P}_1 , \mathcal{P}_2 , and \mathcal{P}'_2 be defined as follows:

- $\mathcal{P}_1 = \{\{(U, L), (U, R)\}, \{(D, L), (D, R)\}\}$,
- $\mathcal{P}_2 = \{\{(U, L)\}, \{(U, R)\}, \{(D, L)\}, \{(D, R)\}\}$, and
- $\mathcal{P}'_2 = \{\{(U, L), (D, L)\}, \{(U, R), (D, R)\}\}$.

Then, \mathcal{P}'_2 is more limited than \mathcal{P}_2 . The set of outcomes of $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in \{1,2\}}, \varphi^{\text{con}} \rangle$ is $\{(U, R), (D, L)\}$, while that of $\Gamma = \langle G, d, \rho, (\mathcal{P}_1, \mathcal{P}'_2), \varphi^{\text{con}} \rangle$ is $\{(U, R), (D, L)\}$ is $\{(U, R), (D, L), (U, L)\}$. Since $u_1(U, L) < \min\{u_1(U, R), u_1(D, L)\}$ this means that it is possible for a player's specifiability to become more limited and the SPE set expands in the direction where the opponent becomes worse off. \square

Now, we are ready to state the comparative statistics result with respect to specifiability.

Corollary 6. *Fix two two-player negotiation games $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$ and $\Gamma' = \langle G, d, \rho', (\mathcal{P}'_i)_{i \in N}, \varphi^{\text{con}} \rangle$ such that ρ and ρ' are asynchronous, $(\mathcal{P}_i)_{i \in N}$ is unlimited and $(\mathcal{P}'_i)_{i \in N}$ is limited. If an alternative $x \in X$ is sustained as a SPE outcome of Γ , then it is sustained as a SPE outcome of Γ' .*

Formally, the proof is a corollary of Theorem 2 and Corollary 3 that we state in Section 4.2, where the former shows that the SPE payoff set under unlimited specifiability is the IR-Preto-meet, and the latter shows that the SPE payoff set under any negotiation rules must include the IR-Pareto-meet.

In the case of limited specifiability, player i saying (No, P_i) is not a commitment to agreeing on an alternative that is an element of P_i . This is because, even if player $-i$ responds with Yes to i 's announcement (No, P_i), player i can always revise her proposal by not announcing (Yes, P_i) again. Of course, this does not mean that there is no commitment under limited specifiability: After i announces (Yes, P_i), by announcing (Yes, P_{-i}) such that $\{x\} = P_i \cap P_{-i}$, player $-i$ can commit to agreeing on the alternative x . For, under the consensual termination rule, if i announces (Yes, P_i) again, players agree on x . This is why the SPE payoff set can be still strictly smaller than the full set even under limited specifiability. In fact, as we have seen, for any negotiation with a common-interest alternative, players' commitment power is so large that the outcome is the unique Pareto-efficient alternative in any SPE of the negotiation game. The exact degree to which such extra scope exists depends critically on the fine detail of the payoff structures as we have seen in Example 4, but the theorem shows that the commitment power cannot be stronger than in the case with unlimited specifiability.

6 Discussions

This section discusses various topics on robustness of our predictions and possible extensions of the framework.

Strategic uncertainty: Strategic uncertainty, as discussed in the Introduction, may play a key role in real negotiations. In Section B.3 of the Online Appendix, we show that the worst Pareto-guaranteeing payoffs are not only the lower bounds of SPE payoffs but also the minimum payoffs achievable even when the order of players' knowledge about the opponent's rationality is limited. Thus, the result demonstrates the relevance of our comparison of SPE payoff sets across different specifiability conditions. In particular, the order of knowledge about rationality necessary for the conclusion of the proposition to hold under each specifiability condition turns out to be very small.

Impatience: We assume no discounting, and one may question the relevance of our results obtained under such an assumption. In Section B.4, we show a continuity result, i.e., when the number of alternatives is finite and there is no tie in payoffs, the set of SPE outcomes under our model with no discounting continues to be the set of SPE outcomes for sufficiently high discount factor strictly less than 1. In light of our discussion in the Introduction in which we argue that impatience may not necessarily play a key role in determining the negotiation outcome in our applications, we believe that our results have economically meaningful content in the applications that we have in mind such as COP meetings where the stakes of the negotiations are high.

Stochastic Announcements: We assume pure strategies in the analysis. In Section B.5 of the Online Appendix, we allow for behavioral strategies, and give an example in which the set of SPE payoff profiles expands. Specifically, it is characterized by the di-convex span of the SPE payoff profiles achievable by pure strategies, which is full-dimensional.³⁰ If the component game corresponds to the Battle of the Sexes and the proposer rule is asynchronous, however, the set of SPE payoff profiles still consists

³⁰See Aumann and Hart (2003) for the definition of di-convex span. This concept is developed in the literature of repeated games with incomplete information (Aumann, Maschler, and Stearns (1968), Aumann and Hart (1986), Forges (1984), and Hart (1985)). See also Forges (1990) on long cheap-talk games, and Forges and Koessler (2008) on long persuasion games.

of two points corresponding to the two strict Nash equilibria.³¹ In particular, no SPE payoff profile gives two players equal payoffs. We view this as formalizing the intuition of Farrell and Rabin (1996). They, in the context of a Battle of the Sexes, question the plausibility of an outcome that assigns equal probability to the two strict Nash equilibria as a consequence of negotiation: “...these fair equilibria do not seem likely to emerge. More plausibly, each player will argue for his or her preferred equilibrium.” We can also establish the robustness of some characterizations of SPE sets. Namely, if U is convex, then X^M continues to be the set of SPE outcomes under asynchronous proposer rule and unlimited specifiability, and $IR(X, d)$ continues to be the set of SPE outcomes under synchronous proposer rule under any specifiability condition.

7 Conclusion

This paper introduced a novel concept that we called limited specifiability, and examined its effect on SPE outcomes. We showed that the effect of limitation on specifiability depends on the move structure. Although there is no difference in the SPE payoff sets under synchronicity, there is a difference when moves are asynchronous. The extent to which such a difference arises can be explained by a tradeoff between commitment and punishment: Limited specifiability necessitates longer periods for reaching a consensus, so there is more scope for punishment. The power of commitment is so strong when the negotiation game has a common-interest alternative that there is a unique SPE outcome under arbitrary specifiability conditions. In order to have all these comparative statics make sense, we defined a negotiation protocol as a collection of three rules, called a proposer rule, a specification rule, and a termination rule. The generality of the model enables us to nest many possible negotiation protocols as special cases of our model, which we believe would facilitate meaningful comparison between different models.

The paper suggests a number of avenues for future research. First, Section B.6 enlists possible variations of proposer, specification and termination rules, demonstrating a wide applicability of our framework to existing models. This suggests that one could use the idea of dividing negotiation protocols to three rules to formally

³¹When the proposer rule is synchronous, on the other hand, such an outcome can be achieved using jointly controlled lotteries à la Aumann, Maschler, and Stearns (1968). Thus the SPE payoff set in this case is a convex hull of the weakly individually rational alternatives. This difference is parallel to the difference between polite and non-polite talks in Aumann and Hart (2003).

compare existing models in the literature with each other. Such an exercise would lead to a unified understanding of the effect of negotiation protocols on the outcomes. Second, there may exist alternative ways to divide negotiation protocols into multiple rules. Our way of dividing it into three rules is just one possibility, and a better justification for our particular choice needs to await further research. Third, our paper is merely a first step in studying limited specifiability. The concept may be applicable in other settings such as cheap talk, delegation, or contracts. Fourth, one could examine whether our prediction is true empirically and/or in experimental settings. For example, Van Huyck, Battalio, and Beil (1990) study the tacit coordination game for which we obtained the unique prediction. Their experimental result seems to roughly match our prediction. Fifth, one could depart from our perfect-information assumption to allow for imperfect/incomplete information. If the limitation on specifiability originated from imperfect information in such a model, then it would be interesting to see how potential resolution of the limitation interacts with incentives of making proposals. Finally, one could imagine endogenizing specifiability, by perhaps introducing costs associated to the degree of specifiability as in the “writing cost” of contracts in Battigalli and Maggi (2002). We hope that our framework of negotiation protocols facilitates unifying the literature, and that more work on limited specifiability will blossom from this paper.

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A Appendix

A.1 Proofs for Section 4

Proof of Theorem 1

Fix a negotiation game with asynchronous proposer rule. Let $x^* \in X$ be a common-interest alternative. Since $x^* \in X^M$, it follows from Theorem 2 in Section 4.2 that x^* can be sustained as a SPE outcome.

Next, we show that x^* is a unique SPE outcome of the negotiation game. Consider the (shortest terminal) history h under which every player is announcing (Yes, P_i), where $\bigcap_{i \in N} P_i = \{x^*\}$. It is without loss of generality to assume that $\rho(h^t) \neq \emptyset$ for any $t \in \{0, \dots, t(h) - 1\}$. At the history $h^{t(h)-1}$, player $i_1 = \rho(h^{t(h)-1})$ can guarantee herself a payoff of $u_{i_1}(x^*)$, her maximum possible SPE payoff of the game (note that any $y \in X$ with $u_i(y) > u_i(x^*)$, if it exists, is not weakly individually rational for some other player). Hence, x^* is the unique outcome in the subgame starting after $h^{t(h)-1}$ in any SPE. Next, at the history $h^{t(h)-2}$, player $i_2 = \rho(h^{t(h)-2})$ can guarantee herself a payoff of $u_{i_2}(x^*)$, her maximum possible SPE payoff of the game. Hence, x^* is the unique outcome in the subgame starting after $h^{t(h)-2}$ in any SPE. Continuing solving backwards in this way, for each $j \in \{1, \dots, t(h)\}$, at any history $h^{t(h)-j}$, player $i_j = \rho(h^{t(h)-j})$ can guarantee herself a payoff of $u_{i_j}(x^*)$, her maximum possible SPE payoff of the game. Hence, x^* is the unique SPE outcome in the subgame starting after the initial history h^0 . That is, the unique SPE outcome of the negotiation game is x^* .

Conversely, suppose that x^* is a unique SPE outcome of the negotiation game. Then, we have $X^M = \{x^*\}$, so that x^* is a unique weakly individually rational and Pareto-efficient alternative. This is because, first, it follows from assumption that $X^* \neq \emptyset$. Second, it follows from Theorem 2 that if X^M were not a singleton set then the negotiation game would have multiple SPE outcomes. Now, for each $i \in N$, her maximum weakly individually rational payoff is $u_i(x^*)$. If not, i.e., if there were $x \in X \setminus \{x^*\}$ such that $u_i(x) = \max_{x' \in \text{IR}(X, d)} u_i(x')$ for some $i \in N$, then $x' \in X^M$, a contradiction. Hence, $u_i(x^*)$ is a unique maximum weakly individually rational payoff for every $i \in N$. Thus, we have $u_i(x^*) > u_i(x)$ for all $x \in \text{IR}(X, d) \setminus \{x^*\}$. This implies that x^* is a common-interest alternative. The proof is complete. \square

Proof of Theorem 2

Fix $x \in X^M$ and denote $x^{(0)} := x$. For each $j \in N$, let $x^{(j)} \in X$ be player j 's worst individually rational and Pareto-efficient alternative. Fix a profile $(P_i^{(j)})_{i \in N}$ of proposals such that $P_i^{(j)} \in \mathcal{P}_i$ for each $i \in N$ and $\{x^{(j)}\} = \bigcap_{i \in N} P_i^{(j)}$, where $j \in \{0\} \cup N$. We denote $P_i = P_i^{(0)}$ for each $i \in N$. Note that it is possible that $x = x^{(j)}$ for some $j \in N$.

Let h^* be the shortest terminal history under which each player i always announces (Yes, P_i). Let $Q_0 := \{h \in H \setminus Z \mid h \sqsubseteq h^*\}$, i.e., Q_0 is the set of non-terminal subhistories of h^* . Next, let Q_j be the set of non-terminal histories under which player $j \in N$ deviates from announcing (Yes, P_j) first. Formally, for each $j \in N$, we let

$$Q_j := \{h \in H_j \setminus Q_0 \mid \min\{i \in N \mid (R_i^{t'}(h), P_i^{t'}(h)) \neq (\text{Yes}, P_i) \text{ for some } t' \in \{1, \dots, t(h)\}\} = j\}.$$

We define the following strategy profile s^* . For each $i \in N$ and $h \in H_i$, we let

$$s_i^*(h) := \begin{cases} (\text{Yes}, P_i) & \text{if } h \in Q_0 \\ s_i^{(j)}(h) & \text{if } h \in Q_j \text{ for some } j \in N \end{cases},$$

where $s_i^{(j)}(h)$ is defined, for any history $h \in H_i \cap Q_j$, as

$$s_i^{(j)}(h) := \begin{cases} (\text{Yes}, \tilde{P}_i) & \text{if } h \in Q_{j,1}^i \\ (\text{Yes}, P_i^{(j)}) & \text{if } h \in Q_{j,2}^i, \\ (\text{No}, P_i^{(j)}) & \text{if } h \in Q_{j,3}^i \end{cases},$$

where the set $H_i \cap Q_j$ is decomposed into the three subsets $Q_{j,1}^i$, $Q_{j,2}^i$, and $Q_{j,3}^i := (H_i \cap Q_j) \setminus (Q_{j,1}^i \cup Q_{j,2}^i)$. The set $Q_{j,1}^i$ contains any non-terminal history h in $H_i \cap Q_j$ with the following properties: (i) some players have already been ok with an alternative \tilde{x} at h ; and (ii) there is a sequence of players and proposals such that it is of each corresponding player's best interest to agree on \tilde{x} . Note that the set $Q_{j,1}^i$ could be

empty. Formally, we let

$$Q_{j,1}^i := \left\{ h \in H_i \cap Q_j \mid \begin{array}{l} \text{there is } \tilde{h}(k^*) := ((N_k, ((\text{Yes}, \tilde{P}_\ell))_{\ell \in N_k}))_{k=1}^{k^*} \text{ with } k^* \geq 1 \\ \text{such that } N_{\ell+1} = \rho(h, \tilde{h}(\ell)) \text{ for each } \ell \in \{0, \dots, k^* - 1\}, \\ \varphi^{\text{con}}(h, \tilde{h}(k^*)) = \tilde{x} \in X \setminus \{x^{(j)}\}, \text{ and } u_\ell(\tilde{x}) > u_\ell(x^{(j)}) \text{ for all } \ell \in \bigcup_{k=1}^{k^*} N_k \end{array} \right\}.$$

Note that $s_i^{(j)}(h) = (\text{Yes}, \tilde{P}_i)$ ($h \in Q_{j,1}^i$) is chosen so that \tilde{P}_i is consistent with a sequence $\tilde{h}(k^*) := ((N_k, ((\text{Yes}, \tilde{P}_\ell))_{\ell \in N_k}))_{k=1}^{k^*}$ such that $\varphi^{\text{con}}(h, \tilde{h}(k^*)) = \tilde{x} \in X \setminus \{x^{(j)}\}$ with $i \in N_1$. Observe that since $u(x^{(j)}) \in \text{WP}(U)$, for any choice of such sequence $\tilde{h}(k^*)$, the set $N \setminus (\bigcup_{k=1}^{k^*} N_k)$ is not empty. Also, the player(s) in this non-empty set have to be ok with \tilde{x} at h , which uniquely pins down \tilde{x} .³² Hence, for any choice of such a sequence $\tilde{h}(k^*)$, the consensual rule uniquely returns $\tilde{x} = \varphi^{\text{con}}(h, \tilde{h}(k^*))$. Henceforth in this proof, we denote by $\tilde{x}(h)$ the unique alternative determined by $h \in Q_{j,1}^i$.

The set $Q_{j,2}^i$ contains any non-terminal history h in $(H_i \cap Q_j) \setminus Q_{j,1}^i$ with the following properties: (i) every player ℓ who spoke at the end of h announced $(\text{No}, P_\ell^{(j)})$; or (ii) every player ℓ has been announcing $(\text{Yes}, P_\ell^{(j)})$ since the most recent announcement of No at time $t^{\text{No}}(h) \leq t(h) - 1$. Formally, we let

$$Q_{j,2}^i := \left\{ h \in (H_i \cap Q_j) \setminus Q_{j,1}^i \mid h = \left(h^{t(h)-1}, (I^{t(h)}(h), ((\text{No}, P_\ell^{(j)}))_{\ell \in I^{t(h)}(h)}) \right) \text{ or} \right. \\ \left. h = \left(h^{t^*}, ((I^k(h), ((\text{Yes}, P_\ell^{(j)}))_{\ell \in I^k(h)}))_{k=t^*+1}^{t(h)} \right) \text{ with } t(h) - 1 \geq t^* := t^{\text{No}}(h) \right\}.$$

We show that player $i \in N$ following s_i^* is a best response to s_{-i}^* in any subgame. Specifically, we find the maximum possible payoff that player $i \in N$ can obtain in a subgame starting after each history, given that any other player $k \in N \setminus \{i\}$ follows s_k^* . At the same time, we show that the strategy profile s^* indeed induces the outcome that attains the maximum payoff that each player can obtain in each subgame. Fix $i \in N$. Consider the following two cases.

Case 1. In the subgame starting from $h \in H_i \cap Q_0$, the maximum payoff that player i can obtain against s_{-i}^* is $u_i(x)$.

Case 2. In any subgame starting from $h \in H_i \cap Q_j$ for some $j \in N$, the maximum

³²Note that \tilde{x} might not be weakly individually rational for some of these players.

payoff that player i can obtain against s_{-i}^* is $u_i(\tilde{x}(h))$ if $h \in Q_{j,1}^i$ and $u_i(x^{(j)})$ otherwise.

Consider Case 2. Fix $j \in N$. Suppose first that $h \in (H_i \cap Q_j) \setminus Q_{j,1}^i$. If player i announces No at a history $h'(\supseteq h)$ at which it is her turn to speak, then, after any subgame starting after h' , any alternative x' with $u_i(x') > u_i(x^{(j)})$ cannot be a SPE outcome. First, she cannot terminate the negotiation with outcome x' at h' . Second, suppose to the contrary that some alternative $x' = \varphi^{\text{con}}(h'')$ with $u_i(x') > u_i(x^{(j)})$ is a SPE outcome with $h'' \supset h'$. Since $u(x^{(j)}) \in \text{WP}(U)$, there is $k \in N \setminus \{i\}$ such that $u_k(x') \leq u_k(x^{(j)})$ and that k is ok with x' at h'' . This is impossible (k is not ok with x' at h'') because such player k , who follows s_k^* , must have said No (at a history at which she speaks in the subgame starting after h' before h''). Hence, the consensual termination rule never returns $x^{(j)}$ in the subgame starting after player i announces No at a history $h'(\supseteq h)$.

Now, for any player i 's strategy such that she announces Yes after each history at which it is her turn to move in the the subgame starting after h , then either every player k keeps announcing (Yes, $P_k^{(j)}$) to agree upon $x^{(j)}$ or some player announces No in the subgame after h . Up to the point at which some player says No in the subgame after h , in the subgame after the next period, any alternative x' with $u_i(x') > u_i(x^{(j)})$ cannot be a SPE outcome. This is because any player $k \in N$ with $u_k(x') \leq u_k(x^{(j)})$ cannot be ok with x' at any subsequent history.

If, on the other hand, player i follows s_i , the strategy profile s^* induces the outcome $x^{(j)}$. Thus, in any subgame starting from $h \in (H_i \cap Q_j) \setminus Q_{j,1}^i$, following s_i^* is a best response to s_{-i}^* .

Second, consider $h \in Q_{j,1}^i$. If player i chooses to follow $s_i^*|_h$, then the strategy profile $s^*|_h$ induces the outcome $\tilde{x}(h)$. Suppose, on the other hand, player i deviates at a history $h'(\supseteq h)$ at which it is her turn to speak. If player i 's announcement was No at that history h' , then any alternative x' with $u_i(x') > u_i(x^{(j)})$ (which, of course, includes $\tilde{x}(h)$) cannot be an outcome of the negotiation game upon such deviation at h' , because any player $k \in N$ with $u_k(x') \leq u_k(x^{(k)})$ cannot be ok with x' at any history in the resulting subgame. If her announcement was Yes, then either every player k keeps announcing (Yes, \tilde{P}_k) to agree upon $\tilde{x}(h)$ or some player announces No at some point. Up to the point at which some player says No, in the subgame after the next period, any induced history at which player i speaks does no longer belong to $Q_{j,1}^i$. Thus, the maximum payoff that player i can obtain against s_{-i}^* in the

subgame following $h \in Q_{j,1}^i$ is $u_i(\tilde{x}(h))$.

Now, we turn to Case 1. Consider the subgame starting after $h \in Q_0 \cap H_i$. If she follows the strategy s_i^* , then the strategy profile $s^*|_h$ induces h^* . This brings her a payoff of $u_i(x)$. If she deviates, then she cannot terminate the game by herself with any other alternative $x' \in X \setminus \{x\}$, because any other player is either ok with x or not ok with any alternative. If her deviation terminates the game with the alternative x , then she obtains a payoff of $u_i(x)$. If not, her deviation induces a non-terminal history $h' = (h, (R_i, P_i)) \in Q_i$. Since no history belonging to $\bigcup_{k \in N \setminus \{i\}} Q_{i,1}^k$ is induced after h' when every $k \in N \setminus \{i\}$ follows s_k^* , player i 's maximum possible payoff in the subgame starting after h' is $u_i(x^{(i)}) (\leq u_i(x))$. Thus, we conclude that player i 's maximum payoff against s_{-i}^* in the subgame starting from the history $h \in Q_0 \cap H_i$ is $u_i(x)$, which is achieved by following s_i^* .

Thus, s_i^* is a best response to s_{-i}^* in any subgame, and hence the strategy profile s^* is a SPE. The SPE s^* induces the history h^* and the outcome x . \square

Remark to the proof of Corollary 3

Fix $i \in N$. Choose $x^{(-i)} \in \operatorname{argmin} \{v_{-i} \mid v \in \operatorname{IR}(U, d) \text{ and } v_i \geq v^{[i, M]}\}$, and hence we have $u_i(x^{(-i)}) = v^{[i, M]}$ and $u_{-i}(x^{(-i)}) = v^{[-i, m]}$.

Now, we show that $v^{[-i, m]}$ is player $-i$'s least weakly individually rational and Pareto-efficient payoff. First, observe that $u(x^{(-i)}) \in \operatorname{WP}(U)$, as any $x \in X$ with $u_i(x) > u_i(x^{(-i)})$ must satisfy $u_{-i}(x) < d_i \leq u_{-i}(x^{(-i)})$. Second, if $v^{[-i, m]}$ were not player $-i$'s least weakly individually rational and Pareto-efficient payoff, there is an alternative $x \in X^M$ with $u_{-i}(x) < u_{-i}(x^{(-i)}) = v^{[-i, m]}$. Since $v^{[-i, m]} = \min\{v_{-i} \mid v \in \operatorname{IR}(U, d) \text{ and } v_i \geq v^{[i, M]}\}$, we must have $u_i(x) < v^{[i, M]}$ (otherwise, $u_{-i}(x) \geq u_{-i}(x^{(-i)}) = v^{[-i, m]}$, a contradiction). Then, $u(x)$ is Pareto-dominated by $u(x^{(-i)})$, a contradiction. Hence, $v^{[-i, m]}$ is player $-i$'s least weakly individually rational and Pareto-efficient payoff. In other words, for each $i \in N$, we can choose $x^{(-i)} \in X$ so that

$$u_i(x^{(-i)}) = \max_{v \in \operatorname{IR}(U, d) \cap \operatorname{WP}(U)} v_i \text{ and } u_{-i}(x^{(-i)}) = \min_{v \in \operatorname{IR}(U, d) \cap \operatorname{WP}(U)} v_{-i}.$$

\square

Proof of Proposition 3

Choose $x^{[i,0]} \in X$ such that $u_i(x^{[i,0]}) = v^{[i,M]}$ and $u_{-i}(x^{[i,0]}) = v^{[-i,m]}$. Also, choose $x^{[i,1]} \in X$ such that $u_i(x^{[i,1]}) = \underline{u}_i$ and $u_{-i}(x^{[i,1]}) \geq v^{[-i,m]}$.

Now, choose a profile of proposals $(P_j^{[i,0]})_{j \in N}$ such that $\{x^{[i,0]}\} = P_1^{[i,0]} \cap P_2^{[i,0]}$ for each $i \in N$. Observe first that at a non-terminal history $h^t = (h^{t-1}, (R_i, P_i^{[i,0]}))$ with $t \in \mathbb{N}$ and $R_i \in \{\text{Yes}, \text{No}\}$, player $-i$ can guarantee herself a payoff of $v^{[-i,m]} = u_{-i}(x^{[i,0]})$ by choosing the announcement (Yes, $P_{-i}^{[i,0]}$) at the history h^t . If $h^{t+1} = (h^t, (\text{Yes}, P_{-i}^{[i,0]}))$ is a terminal history (i.e., $h^t = (h^{t-2}, (R_{-i}, P_{-i}^{[i,0]}), (\text{Yes}, P_i^{[i,0]}))$), then player $-i$ receives a payoff of $v^{[-i,m]} = u_{-i}(x^{[i,0]})$. If not, player i can obtain her maximum possible SPE payoff $v^{[i,M]} = u_i(x^{[i,0]})$ by announcing (Yes, $P_i^{[i,0]}$) after h^{t+1} . Put differently, in the subgame starting after the history $h^t = (h^{t-1}, (R_i, P_i^{[i,0]}))$ with $t \in \mathbb{N}$, a SPE outcome (in this subgame) lies in the set $\{x \in X \mid u(x) \in \text{IR}(U, d) \text{ and } u_{-i}(x) \geq v^{[-i,m]}\}$.

Second, at any history h where it is player i 's turn to move, player i can guarantee a payoff of \underline{u}_i by choosing $(R_i, P_i^{[i,0]})$ at the history h . Hence, the lower bound of player i 's utility in the negotiation game is \underline{u}_i . \square

In words, in a two-player negotiation game, each player i can guarantee herself the minimum payoff in the set of alternatives which yield her opponent (player $-i$) at least as high as her opponent's (player $-i$'s) minimum weakly individually rational and weakly Pareto-efficient payoff.

A.2 Proofs for Section 5

A.3 Proof of Theorem 3: The ‘‘If’’ Direction

As explained in the main text, the proof consists of two steps.

First Step: Suppose to the contrary that, for each i , every $x \in X$ such that $u_i(x) < v^{[i,m]}$ and $u_{-i}(x) \geq v^{[-i,m]}$ is unilaterally improvable for i , but $X^{\text{SPE}} \setminus X^M$ is nonempty. Suppose that $Y := (X^{\text{SPE}} \setminus X^M) \cap \{x \in X \mid u_1(x) < v^{[1,m]}, u_2(x) \geq v^{[2,m]}\}$ is nonempty.

Pick an arbitrary $w \in Y$ such that there is no $w' \in Y$ with $u_1(w) > u_1(w')$ and $u_2(w) < u_2(w')$. We show that w cannot be a SPE outcome. To see this, suppose that there exists a SPE $s \in S$ whose outcome is w .

Fix a non-terminal history \tilde{h} such that $s|_{\tilde{h}}$ induces the outcome w . Since $w \in X$,

$s|_{\bar{h}}$ induces a finite terminal history h . Let h' be a subhistory of h such that $\rho(h') = 1$ and there is no other subhistory \hat{h} with $h' \sqsubset \hat{h} \sqsubset h$ with $\rho(\hat{h}) = 1$.

Let h'' be the unique history such that $h' = (h'', (2, s_2(h'')))$. Let $s_2(h'') = (R_2, P_2)$ and $s_1(h') = (R_1, P_1)$ for $R_1, R_2 \in \{\text{Yes}, \text{No}\}$. By the definition of h' and the consensual termination rule, we have $P_1 \cap P_2 = \{w\}$. Since w is unilaterally improvable by assumption, there exists $w' \in X$ and $P'_1 \in \mathcal{P}_1$ such that $u(w') > u(w)$ and $P'_1 \cap P_2 = \{w'\}$. Pick one such P'_1 .

Consider player 1's deviation to announce (Yes, P'_1) at h' . In the subgame that starts with this history $(h', (\text{Yes}, P'_1))$, one strategy player 2 can take is to announce (Yes, P_2) forever after. By the definition of the consensual termination rule, the negotiation under such a strategy profile terminates at the history $(h', (\text{Yes}, P'_1), (\text{Yes}, P_2))$ with the outcome w' . This implies that in any SPE, player 2's payoff conditional on the history $(h', (\text{Yes}, P'_1))$ is at least $u_2(w')$.

Also, since s is a SPE, her deviation to announcing (Yes, P'_1) cannot lead to a payoff strictly higher than $u_1(w)$. Since $u_2(w') > u_2(w)$, these facts imply that $s|_{\bar{h}}$ leads to an outcome in $\{y \in X | u_1(y) \leq u_1(w) \text{ and } u_2(y) > u_2(w)\}$ after the subgame that starts with this history $(h', (\text{Yes}, P'_1))$.

The above procedure defines, for any nonterminal history h , an infinite sequence (y^1, y^2, \dots) such that, for each $k = 1, 2, \dots$, (i) $y^{k+1} \in \{y \in X | u_1(y) \leq u_1(y^k) \text{ and } u_2(y) > u_2(y^k)\}$ and (ii) there exists \bar{h}^k such that $s|_{\bar{h}^k}$ induces y^k while player 1 has a deviation to announcing (R_1^k, P_1^k) such that $s|_{(\bar{h}^k, (R_1^k, P_1^k))}$ leads to y^{k+1} .

Now, notice that $\{y \in X | u_1(y) \leq u_1(y^{k+1}) \text{ and } u_2(y) > u_2(y^{k+1})\} \subset \{y \in X | u_1(y) \leq u_1(y^k) \text{ and } u_2(y) > u_2(y^k)\}$ and $y^k \notin \{y \in X | u_1(y) \leq u_1(y^k) \text{ and } u_2(y) > u_2(y^k)\}$. Hence, $y^k = \tilde{y}$ implies that there is no $k' > k$ such that $y^{k'} = \tilde{y}$. This contradicts the finiteness of X and the fact that the sequence (y^1, y^2, \dots) is infinite.

Finally, the case in which $Y := (X^{\text{SPE}} \setminus X^M) \cap \{x \in X | u_2(x) < v^{[2,m]}, u_1(x) \geq v^{[1,m]}\}$ is nonempty leads to a contradiction in a symmetric manner.

Second Step: Pick x with $u(x) < (v^{[1,m]}, v^{[2,m]})$ and $u(x) \geq \underline{u}$. Suppose, to the contrary, that there exists a strategy profile s that supports x as a SPE outcome. Let h be the terminal history induced by s .

Let h' be a subhistory of h such that $\rho(h') = 1$ and there is no other subhistory \hat{h} with $h' \sqsubset \hat{h} \sqsubset h$ with $\rho(\hat{h}) = 1$. Let $s_1(h') = (\text{Yes}, P_1)$.³³ Let h'' be the unique history

³³Note that the response is Yes because \mathcal{P}_1 is limited.

such that $h' = (h'', (2, s_2(h'')))$, where $s_2(h'') = (\cdot, P_2)$. Note that $P_1 \cap P_2 = \{x\}$. Pick P'_1 such that there exists x' such that $u(x') = (v^{[1,M]}, v^{[2,m]})$ and $x' \in P'_1$.

Consider a history $\tilde{h} := (h', (1, (\text{No}, P'_1)))$. Under \tilde{h} , if player 2 announces (Yes, P'_2) such that $P'_1 \cap P'_2 = \{x'\}$ with $u(x') = (v^{[1,M]}, v^{[2,m]})$, then player 1 can receive the best individually rational payoff $v^{[1,M]}$. This is the best payoff player 1 can receive under the subgame starting at \tilde{h} and the continuation play by player 2 follows $s_2|_{\tilde{h}}$. This is because if player 1 announces P''_1 such that $P''_1 \cap P'_2 \neq \{x'\}$, then the negotiation does not terminate and player 2 then has a strategy to respond with (No, P'_2) indefinitely, which means player 2 must receive an individually rational payoff under \tilde{h} . Hence, player 1's payoff under $\tilde{h} = (h', (1, (\text{No}, P'_1)), (2, (\text{Yes}, P'_2)))$ is $v^{[1,M]}$.

Given that player 1 receives $v^{[1,M]}$, feasibility requires that player 2 receives payoff no less than $v^{[2,m]}$. Hence, $s|_{\tilde{h}}$ must induce an outcome that gives player 2 a payoff no less than $v^{[2,m]}$.

Next, since $P'_1 \neq P_1$, by the equilibrium condition, $s_{\tilde{h}}$ must induce an outcome that gives player 1 a payoff no greater than $u_1(x)$.

Overall, letting the outcome induced by $s|_{\tilde{h}}$ be z , $u_1(z) \leq u_1(x)$ and $u_2(z) \geq v^{[2,m]}$. Since $u_1(x) < v^{[1,m]}$, we have $u_1(z) < v^{[1,m]}$ and $u_2(z) \geq v^{[2,m]}$. Letting $y^1 = z$, one must be able to construct an infinite sequence defined in the first step, which leads to a contradiction. \square

A.4 Proof of Theorem 3: The “Only If” Direction (The Proof of Proposition 5)

Proof. The proof consists of two steps. In the first step, we show that for each $i \in N$, any alternative $x \in X$ such that (1-i) $u_i(x) < v^{[i,m]}$ and $u_{-i}(x) \geq v^{[-i,m]}$, and (1-ii) x is not unilaterally improvable for player i , can be sustained as a SPE for each $i \in N$. In the second step, we show that any alternative $x \in X$ such that (2-i) $\underline{u}_i \leq u_i(x) < v^{[i,m]}$ for each $i \in N$, (2-ii) x is not unilaterally improvable for both players, and (2-iii) there exists $(y^1, y^2) \in X^2$ such that, for each $i \in \{1, 2\}$, $u_i(y^i) \leq u_i(x)$ and $u_{-i}(y^i) \geq v^{[-i,m]}$ and y^i is not unilaterally improvable for player i , can be sustained as a SPE.

Step 1. Fix $i \in N$ and let $x \in X$ be an alternative satisfying (1-i) and (1-ii). Fix $(P_1, P_2) \in \mathcal{P}_1 \times \mathcal{P}_2$ such that $\{x\} = P_1 \cap P_2$ and that there is no $(P'_i, y) \in \mathcal{P}_i \times X$ with $\{y\} = P'_i \cap P_{-i}$ and $u(y) > u(x)$. Let $y^{-i} \in X$ be an alternative such that

$u_{-i}(y^{-i}) = v^{[-i,m]}$ and $u_i(y^{-i}) = v^{[i,M]}$. Choose a profile of proposals $(\tilde{P}_1, \tilde{P}_2)$ such that $\tilde{P}_1 \cap \tilde{P}_2 = \{y^{-i}\}$.

We divide the set of non-terminal histories $H \setminus Z$ into the two sets: Q_i and $Q_{-i} := (H \setminus Z) \setminus Q_i$. We let Q_i be the set of non-terminal histories in which player $-i$ always responds with (i) (Yes, P_{-i}) to (Yes, P_i) and (ii) (No, P_{-i}) otherwise. Formally, we define

$$Q_i := \left\{ h \in H \setminus Z \mid \text{for any } h^t \text{ with } h^t \sqsubset h \text{ and } \rho(h^t) = -i, \right. \\ \left. h^{t+1} \in \{(h^{t-1}, (R'_i, P'_i), (\text{No}, P_{-i})), (h^{t-1}, (\text{Yes}, P_i), (\text{Yes}, P_{-i}))\}, \text{ where } (R'_i, P'_i) \neq (\text{Yes}, P_i) \right\}.$$

Now, consider the following strategy profile s^* . For player i , for any $h \in H_i$,

$$s_i^*(h) = \begin{cases} (\text{Yes}, P_i) & \text{if } h \in Q_i \\ s_i^{(-i)}(h) & \text{if } h \in Q_{-i} \end{cases},$$

where $s_i^{(-i)}$ is defined on $H_i \cap Q_{-i}$ as follows:

$$s_i^{(-i)}(h) = \begin{cases} (\text{Yes}, \tilde{P}_i) & \text{if } h \in \left\{ (h^{t(h)-1}, (\text{No}, \tilde{P}_{-i})), (h^{t(h)-2}, (\text{No}, \tilde{P}_i), (\text{Yes}, \tilde{P}_{-i})) \right\} \\ (\text{No}, \tilde{P}_i) & \text{otherwise} \end{cases}.$$

Likewise, for player $-i$, we let

$$s_{-i}^*(h) = \begin{cases} (\text{Yes}, P_{-i}) & \text{if } h \in H_{-i} \cap Q_i \text{ and } h = (h^{t(h)-1}, (\text{Yes}, P_i)) \\ (\text{No}, P_{-i}) & \text{if } h \in H_{-i} \cap Q_i \text{ and } h \neq (h^{t(h)-1}, (\text{Yes}, P_i)), \\ s_{-i}^{(-i)}(h) & \text{if } h \in H_i \cap Q_{-i} \end{cases},$$

where $s_{-i}^{(-i)}(h)$ is defined as follows:

$$s_{-i}^{(-i)}(h) = \begin{cases} (\text{Yes}, \tilde{P}_{-i}) & \text{if } h = \left(h^{t(h)-2}, (R_{-i}, \tilde{P}_{-i}), (\text{Yes}, \tilde{P}_i) \right) \text{ with } t(h) \geq 2, \\ & u_{-i}(\tilde{x}) > u_{-i}(x^{[i,0]}), \text{ and } \{\tilde{x}\} = \tilde{P}_1 \cap \tilde{P}_2 \\ (\text{Yes}, P_{-i}^{-i}) & \text{if } h \in \left\{ (h^{t(h)-1}, (\text{No}, P_i^{-i})), (h^{t(h)-2}, (\text{No}, P_{-i}^{-i}), (\text{Yes}, P_i^{-i})) \right\} \\ (\text{No}, P_{-i}^{-i}) & \text{otherwise} \end{cases}.$$

By inspection one can check that player $j \in N$ following s_j^* is a best response to s_{-j}^* in any subgame.³⁴

Step 2. Fix an alternative $x \in X$ with $(v^{[1,m]}, v^{[2,m]}) > u(x) \geq \underline{u}$, which satisfies the conditions (2-i)-(2-iii). Fix a profile of proposals $(P_j)_{j \in N}$ such that $\{x\} = P_1 \cap P_2$ such that for each i , there is no $(P'_{-i}, y) \in \mathcal{P}_{-i} \times X$ with $P_i \cap P'_{-i} = \{y\}$ and $u(y) > u(x)$.

Fix a pair $(y^1, y^2) \in X^2$ satisfying (2-iii). For each $i \in \{1, 2\}$, fix proposals $(P_j^i)_{j \in N}$ such that (a) $\{y^i\} = P_1^i \cap P_2^i$ and (b) there is no $(P'_i, z) \in \mathcal{P}_i \times X$ with $P'_i \cap P_{-i}^i = \{z\}$ and $u(z) > u(y^i)$.

First, we decompose the set of non-terminal histories $H \setminus Z$ into the following sets in two steps. As a first step, we divide the set H into the following three sets. First, let $Q_0 := \{h^0, (\text{Yes}, P_1), ((\text{Yes}, P_1), ((\text{Yes}, P_2)))\}$. Second, let Q_i be the set of non-terminal histories under which player i deviates from announcing (Yes, P_i) first. Formally, we let

$$Q_2 := \{h \in (H \setminus Z) \setminus Q_0 \mid h^1 = ((\text{Yes}, P_1)) \text{ and } h^2 \neq ((\text{Yes}, P_1), ((\text{Yes}, P_2)))\}$$

and $Q_1 := (H \setminus Z) \setminus (Q_0 \cup Q_2)$.

We further divide the sets of non-terminal histories Q_1 and Q_2 as follows.

$$Q_i^{\text{on}} := \left\{ h \in Q_i \mid \begin{array}{l} \text{for any } t < t(h) \text{ such that } \rho(h^t) = -i \text{ and } h^t \notin Q_0, \\ h^{t+1} = (h^{t-1}, (R_i^t(h), P_i^i), (\text{Yes}, P_{-i}^i)) \text{ or } h^{t+1} = (h^{t-1}, (R_i^t(h), P'_i), (\text{No}, P_{-i}^i)) \\ \text{for some } P'_i \in \mathcal{P}_i \setminus \{P_i\} \end{array} \right\}.$$

and $Q_i^{\text{off}} := Q_i \setminus Q_i^{\text{on}}$.

Now, we define the strategy profile s^* which sustains x . Player i 's strategy is defined as follows.

- At any history $h \in H_i \cap Q_0$, let $s_i^*(h) := (\text{Yes}, P_i)$.
- At any history $h \in H_i \cap Q_i^{\text{on}}$, let $s_i^*(h) = (\text{Yes}, P_i^i)$.

³⁴For completeness, in Online Appendix B.10, we explicitly check the best response condition.

- At any history $h \in H_i \cap Q_{-i}^{\text{on}}$, let

$$s_i^*(h) := \begin{cases} (\text{Yes}, P_i^{-i}) & \text{if } h = \left(h^{t(h)-1}, (R_{-i}^{t(h)}(h), P_{-i}^{-i}) \right) \\ (\text{No}, P_i^{-i}) & \text{otherwise} \end{cases}.$$

- At any history $h \in H_i \cap Q_i^{\text{off}}$, we let

$$s_i^*(h) := \begin{cases} (\text{Yes}, \tilde{P}_i^{-i}) & \text{if } h \in \left\{ \left(h^{t(h)-1}, (\text{No}, \tilde{P}_{-i}^{-i}) \right), \left(h^{t(h)-2}, (R_i^{t(h)-1}(h), \tilde{P}_i^{-i}), (\text{Yes}, \tilde{P}_{-i}^{-i}) \right) \right\} \\ (\text{No}, \tilde{P}_i^{-i}) & \text{otherwise} \end{cases},$$

where a profile of proposals $(\tilde{P}_1^{-i}, \tilde{P}_2^{-i}) \in \mathcal{P}_1 \times \mathcal{P}_2$ is defined so that $\tilde{P}_1^{-i} \cap \tilde{P}_2^{-i} = \{y^{-i}\}$, $u_{-i}(y^{-i}) = v^{[-i,m]}$, and $u_i(y^{-i}) = v^{[i,M]}$.

- At any history $h \in H_i \cap Q_{-i}^{\text{off}}$, we let

$$s_i^*(h) = \begin{cases} (\text{Yes}, \hat{P}_i) & \text{if } h = \left(h^{t(h)-2}, (R_i^{t(h)-1}(h), \hat{P}_i), (\text{Yes}, \hat{P}_{-i}) \right) \\ & \text{with } u_i(\hat{x}) > v^{[i,m]} \text{ and } \{\hat{x}\} = \hat{P}_1 \cap \hat{P}_2 \\ (\text{Yes}, \tilde{P}_i^i) & \text{if } h \in \left\{ \left(h^{t(h)-1}, (\text{No}, \tilde{P}_{-i}^i) \right), \left(h^{t(h)-2}, (R_i^{t(h)-1}, \tilde{P}_i^i), (\text{Yes}, \tilde{P}_{-i}^i) \right) \right\} \\ (\text{No}, \tilde{P}_i^i) & \text{otherwise} \end{cases},$$

where a profile of proposals $(\tilde{P}_1^i, \tilde{P}_2^i) \in \mathcal{P}_1 \times \mathcal{P}_2$ is defined so that $\tilde{P}_1^i \cap \tilde{P}_2^i = \{y^i\}$, $u_i(y^i) = v^{[i,m]}$, and $u_{-i}(y^i) = v^{[-i,M]}$.

By inspection one can check that player $i \in N$ following s_i^* is a best response to s_{-i}^* in any subgame.³⁵ \square

Proof of Corollary 5

Let $X = U^2$. Suppose that the specification rule is given by $\mathcal{P}_i = \{\{u\} \times U \mid u \in U\}$. Each player i 's payoff is given by $u_i(v, v') = v_i$ if $v = v'$ and $u_i(v, v') = d_i$ if $v \neq v'$.

³⁵Again, for completeness, in Online Appendix B.10, we explicitly check the best response condition.

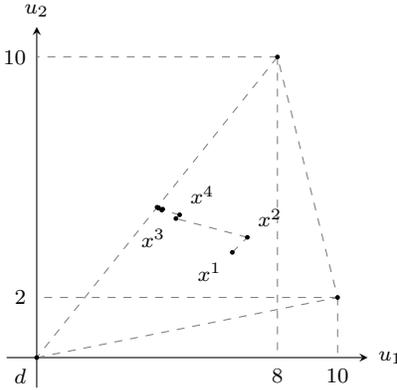


Figure B.1: Counterexample to the “If” direction of Theorem 3 for infinite X

B Online Appendix (Not for Publication)

B.1 Counterexample to the “If” Direction of Theorem 3 for Infinite X

Define $X = \{(10, 2), (8, 10), (4, 5), (0, 0)\} \cup \{x^n \mid n \in \mathbb{N}\}$ where

$$x^n = \begin{cases} (4 + (\frac{1}{2})^{n-2}, 5 - (\frac{1}{2})^{n-2}) & \text{if } n \text{ is odd} \\ (4 + 3(\frac{1}{2})^{n-2}, 5 - 5(\frac{1}{2})^n) & \text{if } n \text{ is even} \end{cases}.$$

Let the disagreement payoffs be $d = (0, 0)$. We define players’ payoff functions by $u_i(x) = x_i$ for each $x \in X$ and $i \in N$. Note that $\text{IR}(U, d)$ is compact. The specification rules are defined as:

$$\begin{aligned} \mathcal{P}_1 &= \{\{x^n, (0, 0)\} \mid n \in \mathbb{N}\} \cup \{\{(10, 2), (4, 5)\}, \{(8, 10), (0, 0)\}\}; \text{ and} \\ \mathcal{P}_2 &= \{\{x^{2n-1}, x^{2n}\} \mid n \in \mathbb{N}\} \cup \{\{x^{2n}, (8, 10)\} \mid n \in \mathbb{N}\} \\ &\quad \cup \{\{(10, 2), (8, 10)\}, \{(8, 10), (4, 5)\}, \{(4, 5), (0, 0)\}\}. \end{aligned}$$

Notice that $u_1(x^1) < v^{[1,m]}$ and $u_2(x^1) \geq v^{[2,m]}$. Also, x^1 is unilaterally improvable for player 1 because the only pair (P_1, P_2) such that $P_1 \cap P_2 = \{x^1\}$ is $(P_1, P_2) = (\{x^1, (0, 0)\}, \{x^1, x^2\})$, and $P'_1 = \{x^2, (0, 0)\}$ has a property that $P'_1 \cap P_2 = \{x^2\}$ and $u(x^2) > u(x^1)$.

We construct a SPE s that induces x^k with $k = 2n - 1$ for some $n \in \mathbb{N}$ as an

outcome.

The idea of the construction is as follows. Consider sustaining x^k . First, any deviation by player 2 is punished by the outcome $(10, 2)$. In order to incentivize player 1 to comply with the specified strategy, we define a sequence of punishments. If player 1 deviates when the game is supposed to end with outcome x^K under a given history, then players' future strategies are such that the game ends with outcome x^{K+2} . This specification provides player 1 an appropriate incentive for any finite length of histories because there are infinitely many alternatives (i.e., for any $K = 2n - 1$ for $n \in \mathbb{N}$, there exists $x^K \in X$).

The strategy profile $s = (s_1, s_2)$ is defined recursively as follows. First, we let $s_1(h^0) = (\text{Yes}, \{x^k, (0, 0)\})$ and for h such that $t(h) = 1$,

$$s_2(h) = \begin{cases} (\text{Yes}, \{x^k, x^{k+1}\}) & \text{if } (R_1^1(h), P_1^1(h)) = (\text{Yes}, \{(10, 2), (0, 0)\}) \\ (\text{Yes}, \{x^{k+2}, x^{k+3}\}) & \text{if } (R_1^1(h), P_1^1(h)) \neq (\text{Yes}, \{(10, 2), (0, 0)\}) \end{cases}.$$

We let $H_i(h) := \{h^t \in H_i \mid h^t \in H_i \text{ and } t < t(h)\}$ for each $i \in N$ and $h \in H \setminus Z$. Suppose that s_1 and s_2 are defined on $H_1(h)$ and $H_2(h)$. We define $s_i(h)$ in what follows.

First, we specify $s_1(h)$ for each $h \in H_1$ with $t(h) > 1$.

1. If there exists $h^t \in H_2(h)$ such that $s_2(h^t) \neq (R_2^{t+1}(h), P_2^{t+1}(h))$, then
 - (a) if $\varphi^{\text{con}}(h, (\text{Yes}, \{(10, 2), (0, 0)\})) = (0, 0)$, then $s_1(h) = (\text{No}, \{(10, 2), (0, 0)\})$.
 - (b) otherwise, $s_1(h) = (\text{Yes}, \{(10, 2), (0, 0)\})$.
2. Otherwise, $s_1(h) = (\text{Yes}, \{x^{k+2\ell(h)}, (0, 0)\})$ where $\ell(h) := |\{t' \in \mathbb{N}_0 \mid h^{t'} \in H_1(h) \text{ and } s_1(h^{t'}) \neq (R_1^{t'+1}(h), P_1^{t'+1}(h))\}|$.

Next, we specify $s_2(h)$ for each $h \in H_2$ with $t(h) > 1$.

1. If there exists $h^t \in H_2(h)$ such that $s_2(h^t) \neq (R_2^{t+1}(h), P_2^{t+1}(h))$, then
 - (a) if there exists $P_2 \in \mathcal{P}_2$ such that $\varphi^{\text{con}}(h, (\text{Yes}, P_2)) = x$ with $u_2(x) > 2$, then he announces (Yes, P_2) (there exists at most one such P_2).
 - (b) otherwise, he announces $s_2(h) = (\text{Yes}, \{(10, 2), (8, 10)\})$.
2. Otherwise, $s_2(h) = (\text{Yes}, \{x^{k+2\ell(h)}, x^{k+2\ell(h)+1}\})$.

By inspection, one can check that s is a SPE. □

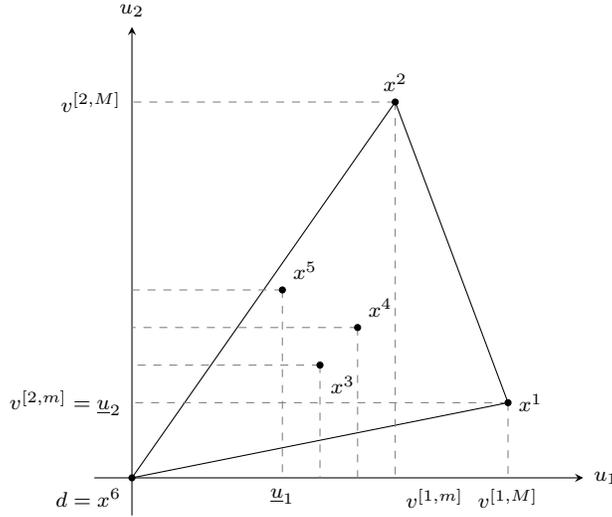


Figure B.2: An example in which an unilateral improvable alternative $x^5 \in X \setminus X^M$ is a SPE outcome.

B.2 An Example in which an Unilateral Improvable Alternative in $X \setminus X^M$ is a SPE Outcome.

Here we provide an example in which, $x \in X \setminus X^M$ with $u(x) \geq \underline{u}$ is unilaterally improvable but is a SPE outcome when there is $y \in X \setminus X^M$ with $u(y) \geq \underline{u}$ that is not unilaterally improvable.

Specifically, let $X = \{x^1, x^2, \dots, x^6\}$ be such that $u(x^1) = (10, 2)$, $u(x^2) = (7, 10)$, $u(x^3) = (5, 3)$, $u(x^4) = (6, 4)$, $u(x^5) = (4, 5)$, and $u(x^6) = (0, 0)$. The disagreement payoffs are $d = (0, 0)$. We define the specification rule as follows:

$$\begin{aligned} \mathcal{P}_1 &= \{\{x^1, x^6\}, \{x^2, x^6\}, \{x^3, x^6\}, \{x^4, x^6\}, \{x^5, x^6\}\}, \\ \mathcal{P}_2 &= \{\{x^1, x^2\}, \{x^3, x^4\}, \{x^1, x^5\}, \{x^4, x^6\}\}. \end{aligned}$$

Note that x^3 is unilaterally improvable for player 1, but for each $k \in \{4, 5\}$, x^k is not unilaterally improvable for player 1. We define a strategy profile (s_1, s_2) together with states θ_1 , θ_3 , and θ_5 and the transition rule among those states as follows.

The initial state is θ_3 . Suppose that the state is θ_3 .

1. Strategies:

(a) $s_1(h) = (\text{Yes}, \{x^3, x^6\})$ for any $h \in H_1$

- (b) If $h = (\dots, (R_1, P_1))$ with $P_1 \neq \{x^3, x^6\}$, then $s_2(h) = (\text{No}, \{x^3, x^4\})$.
 Otherwise, $s_2(h) = (\text{Yes}, \{x^3, x^4\})$. for any $h \in H_2$.

2. State Transition:

- (a) If player 1 announces $(\text{Yes}, \{x^4, x^6\})$, at θ_3 , then the state changes to θ_5 .
 (b) If player 2 does not follow this strategy at θ_3 , the state changes to θ_1 .
 (c) Otherwise the state stays at θ_3 .

Suppose that the state is θ_1 .

1. Strategies:

- (a) Fix $h \in H_1$. If $\varphi^{\text{con}}(h, (\text{Yes}, \{x^1, x^6\})) \neq x^6$ then $s_1(h) = (\text{Yes}, \{x^1, x^6\})$.
 Otherwise, $s_1(h) = (\text{No}, \{x^1, x^6\})$.
 (b) Fix $h \in H_2$. If there exists $P_2 \in \mathcal{P}_2$ such that $\varphi^{\text{con}}(h, (\text{Yes}, P_2)) = x^k$ for
 $k \in \{2, 3, 4, 5, 6\}$, then $s_2(h) = (\text{Yes}, P_2)$. If $\varphi^{\text{con}}(h, (\text{Yes}, \{x^1, x^6\})) = x^6$,
 then $s_2(h) = (\text{No}, \{x^1, x^5\})$. Otherwise, $s_2(h) = (\text{Yes}, \{x^1, x^5\})$.

2. State Transition: The state does not change and stays at θ_1 .

Suppose that the state is θ_5 .

1. Strategies:

- (a) Fix $h \in H_1$. If $\varphi^{\text{con}}(h, (\text{Yes}, \{x^5, x^6\})) \neq x^6$ then $s_1(h) = (\text{Yes}, \{x^5, x^6\})$.
 Otherwise, $s_1(h) = (\text{No}, \{x^5, x^6\})$.
 (b) If $h = (\dots, (R_1, P_1))$ with $P_1 \neq \{x^5, x^6\}$ or $\varphi^{\text{con}}(h, (\text{Yes}, \{x^1, x^5\})) = x^1$,
 then $s_2(h) = (\text{No}, \{x^1, x^5\})$. Otherwise, $s_2(h) = (\text{Yes}, \{x^1, x^5\})$.

2. State Transition:

- (a) If player 2 does not follow this strategy at θ_5 , then the state changes to θ_1 .
 (b) Otherwise the state stays at θ_5 .

The outcome induced by s is x^5 . By inspection, one can check that s is a SPE.

B.3 Strategic Uncertainty

The difference of the SPE payoff sets under the two specifiability conditions lie in the payoffs that are Pareto-dominated by at least some payoffs that are achievable under unlimited specifiability. Without any extra reason to expect efficiency in negotiations, we do not have a reasonable justification for undervaluing the importance of such payoffs. However, still, one may question the practical importance of such payoffs.

Here we argue that the worst Pareto-guaranteeing payoffs are not only the lower bounds of SPE payoffs but also the minimum payoffs achievable even when the order of players' knowledge about the opponent's rationality is limited. As we discussed in the Introduction, strategic uncertainty may play a key role in real negotiations, so the result serves as demonstrating the value in paying attention to the difference between the two specifiability conditions.

Let $h(\bar{s})$ be the random variable that corresponds to the history induced by behavioral strategy profile \bar{s} . Define ordinal preferences \succsim_i over \bar{S} by $\bar{s} \succsim_i \bar{s}'$ if and only if (i) $\bar{u}_i(\bar{s}) > \bar{u}_i(\bar{s}')$ or (ii) $\bar{u}_i(\bar{s}) = \bar{u}_i(\bar{s}')$ and $\mathbb{E}[t(h(\bar{s}))] \geq \mathbb{E}[t(h(\bar{s}'))]$. That is, \succsim_i is a lexicographic preference relation over behavioral strategy profiles that first considers the expected payoff and then the expected time until an agreement.³⁶

We define $\bar{S}_i^{[1]}$ to be the set of i 's behavioral strategies that are best responses to *some* beliefs about the opponent's strategies. Then we define $\bar{S}_i^{[2]}$ to be the set of i 's behavioral strategies that are best responses to some beliefs about the opponent's strategies that are in $\bar{S}_{-i}^{[1]}$. Formally, we let:

$$\begin{aligned} \bar{S}_i^{[1]} &:= \{\bar{s}_i \in \bar{S}_i \mid \exists \bar{s}_{-i} \in \bar{S}_{-i} \text{ such that } (\bar{s}_i, \bar{s}_{-i}) \succsim_i (\bar{s}'_i, \bar{s}_{-i}) \text{ for all } \bar{s}'_i \in \bar{S}_i\}; \text{ and} \\ \bar{S}_i^{[2]} &:= \{\bar{s}_i \in \bar{S}_i \mid \exists \bar{s}_{-i} \in \bar{S}_{-i}^{[1]} \text{ such that } (\bar{s}_i, \bar{s}_{-i}) \succsim_i (\bar{s}'_i, \bar{s}_{-i}) \text{ for all } \bar{s}'_i \in \bar{S}_i\}. \end{aligned}$$

Proposition B.1. *Fix a two-player negotiation game $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$, where ρ is asynchronous.*

1. *If specifiability is unlimited, then v is a SPE payoff profile if and only if for each $i \in N$, $v_i \geq \max_{\bar{s}_i \in \bar{S}_i} \min_{\bar{s}_{-i} \in \bar{S}_{-i}^{[1]}} \bar{u}_i(\bar{s}_i, \bar{s}_{-i})$.*

³⁶We view such preferences for early consensus as plausible in reality so long as early consensus does not lower one's payoff. Technically, without such an assumption, a player may be able to believe that her opponent will not terminate the negotiation even when he can receive the best feasible payoff in a belief that he can do so later with probability one.

2. If specifiability is limited, then v is a SPE payoff profile if and only if for each $i \in N$, $v_i \geq \max_{\bar{s}_i \in \bar{S}_i} \min_{s_{-i} \in S_{-i}^{[2]}} \bar{u}_i(\bar{s}_i, s_{-i})$.

Part 1 of this proposition shows that, under unlimited specifiability, i 's minimum SPE payoff coincides with her maxmin payoff when she believes in $-i$'s rationality. Part 2 then shows that, under limited specifiability, i 's minimum SPE payoff coincides with her maxmin payoff when she believes in $-i$'s rationality and his knowledge about i 's rationality. As we have discussed, these results demonstrate the relevance of our comparison of SPE payoff sets across different specifiability conditions. In particular, the order of knowledge about rationality necessary for the conclusion of the proposition to hold under each specifiability condition turned out to be very small.

B.4 Impatience

Theorem 2 implies that if the IR-Pareto-meet consists of multiple points, then there are multiple SPE alternatives under any proposer and specification rules. This is in a stark contrast to the uniqueness of SPE in many bargaining models with complete information and asynchronous proposer rule, such as Rubinstein (1982) and Ståhl (1972). The reason for this difference is that we do not assume discounting. For the following argument, assume $d_i = 0$ for all $i \in N$.

To see the connection clearly, first note that in Rubinstein's (1982) bargaining model, if the discount factor δ is exactly equal to one and indefinite agreement results in the payoff of zero, then all possible divisions of the pie can be sustained under SPE. A related result is that if one discretizes the space of offers to make it a finite set, then for sufficiently large $\delta < 1$, all possible divisions of the pie can be sustained under SPE (Muthoo (1991) and Van Damme, Selten, and Winter (1990)).

A parallel result can be obtained in our model. Consider a two-player case, and suppose that the proposer rule is asynchronous. If the feasible payoff set is convex and the Pareto-frontier is characterized by a strictly decreasing continuous function, then there is a unique SPE payoff profile under any specification rules for any $\delta < 1$. On the other hand, for any feasible payoff set consisting of a finite number of points with no payoff ties, our characterization of SPE remains unchanged for sufficiently large $\delta < 1$.

Formally, the following result holds. Let $E(\delta)$ be the set of SPE alternatives when the discount factor is δ .

Proposition B.2. Fix $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$ with $d_i = 0$ for all $i \in N$ such that

1. ρ is synchronous, or

2. X is finite, and

(a) Γ has a common interest alternative, or

(b) $N = \{1, 2\}$ and there is no i and no pair of distinct alternatives (x, y) such that $u_i(x) = u_i(y)$.

Then, there exists $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$, $E(\delta) = E(1)$.

The above argument suggests that the relevance of our argument surrounding the comparison of specifiability depends crucially on the relative magnitudes of the salience of impatience and the variety of available alternatives. In light of our discussion in the Introduction in which we argue that impatience may not play a key role in determining the negotiation outcome in our applications, we believe that our results have economically meaningful content in the applications that we have in mind such as COP meetings where the stakes of the negotiations are high.

Finally, we note that under synchronous proposer rules, we indeed have $E(\delta) = E(1)$ for all $\delta \in (0, 1]$.³⁷

B.5 Stochastic Announcements

In this section, we allow each player to make their announcements stochastically. We consider a component game $G = \langle N, X, (u_i)_{i \in N} \rangle$ such that the set $U := \{u(x) \in \mathbb{R}^n \mid x \in X\}$ of feasible payoffs is bounded in \mathbb{R}^n . We also assume that $\mathcal{P}_i \subseteq \mathcal{F}(X) \cup \{\emptyset\}$ for each $i \in N$, where $\mathcal{F}(X)$ is the set of non-empty closed subsets of X .³⁸

For each player $i \in N$, a *behavioral strategy* of player i is a mapping $\bar{s}_i : H_i \rightarrow \Delta(\{\text{Yes, No}\} \times (\mathcal{F}(X) \cup \{\emptyset\}))$ such that $\text{supp}(\bar{s}_i(h)) \subseteq \{\text{Yes, No}\} \times \mathcal{P}_i$.³⁹ Here, $\Delta(\mathcal{X})$

³⁷Stahl (1986) examines a dynamic Bertrand competition model with or without discounting where each seller can synchronously change her price announcements. Analogous to our “folk theorem” here, he shows that any price less than or equal to the monopoly price can be sustained as a SPE outcome.

³⁸First, observe that $\{x\} \in \mathcal{F}(X)$ for all $x \in X$. Second, suppose that the component game G is given by a normal-form game $\langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$, where each A_i is a metric space. If, for example, $\mathcal{P}_i = \{\emptyset\} \cup \{\{a\} \mid a \in A\}$ under the unlimited specification rule and $\mathcal{P}_i = \{\emptyset\} \cup \{\{a_i\} \times A_{-i} \mid a_i \in A_i\}$ under a limited specification rule, the assumption that $\mathcal{P}_i \subseteq \mathcal{F}(X) \cup \{\emptyset\}$ is satisfied.

³⁹Restricting to behavioral strategies entail may entail a certain loss of generality because our extensive-form game involves an infinite number of nodes. See Aumann (1964).

denotes the set of all probability measures on the Borel sets of \mathcal{X} , where \mathcal{X} is a metric space.⁴⁰ Also, $\text{supp}(\chi)$ is the support of χ .

The set of player i 's behavioral/stochastic strategies is denoted by \bar{S}_i . The set of behavioral/stochastic strategy profiles is denoted by $\bar{S} := \times_{i \in N} \bar{S}_i$.

An *outcome* of the negotiation game induced by a profile of behavioral strategies \bar{s} refers to the distribution on the terminal histories Z induced by the strategy profile \bar{s} . A strategy profile \bar{s} uniquely determines an outcome, which in turn uniquely determines the expected payoff that we denote by $\bar{u}_i : \bar{S} \rightarrow \mathbb{R}$.⁴¹

With the above formulation, we can define SPE and the payoff sets associate with SPE. In general, the SPE payoff sets can be quite complicated. We illustrate this point in the following example.

Example B.1. Consider the normal-form game given by the left panel of Figure B.3, and assume an asynchronous proposer rule and limited specifiability. Let $d = (0, 0)$. By inspection, one can show that the set of pure-strategy SPE payoffs is given by $\{(3, 1), (2, 3), (1, 0), (0, 4), (0, 2)\}$. On the other hand, the set of SPE payoffs when players use behavioral strategies is given by the right panel of Figure B.3. Namely, the set of SPE payoffs corresponds to the di-convex span of the set of pure-strategy SPE payoffs.⁴²

For each point in the di-convex span, we construct a SPE to support such a payoff profile. The construction closely follows that of long cheap talk in Aumann and Hart (2003). To understand the idea, consider the payoff profile $(1, 2)$. To sustain this payoff profile, at the initial period player 1 mixes between (No, U) and (No, D) with probability $1/2$ for each, where the former induces the continuation payoff $(1, 1)$ and the latter induces $(1, 3)$. Now, consider sustaining a point on the six solid line-segments in Figure B.3 except for the points in $\{(3, 1), (2, 3), (1, 0), (0, 4), (0, 2)\}$. As an example, take a point $(2, 1)$ on the line segment from $(1, 1)$ to $(3, 1)$. To sustain this continuation payoff profile, player 2 mixes in his turn between (No, L) and (No, R) , where the former is assigned probability $1/2$ and induces the continuation payoff $(1, 1)$, while the latter is assigned probability $1/2$ and induces $(3, 1)$. If $(3, 1)$ is reached, then

⁴⁰Given that X is a metric space, we can introduce the Hausdorff metric topology on $\mathcal{F}(X)$. Then, we add the empty set to $\mathcal{F}(X)$ as an isolated point of $\mathcal{F}(X) \cup \{\emptyset\}$ and consider an appropriate product metric on $\{\text{Yes}, \text{No}\} \times (\mathcal{F}(X) \cup \{\emptyset\})$, where we introduce the discrete metric on $\{\text{Yes}, \text{No}\}$.

⁴¹The expectation is well-defined because u_i is bounded and Z is a measurable space.

⁴²See Aumann and Hart (2003) for the definition of di-convex span. Indeed, the right panel of Figure B.3 coincides with Aumann and Hart (2003, Figure 10).

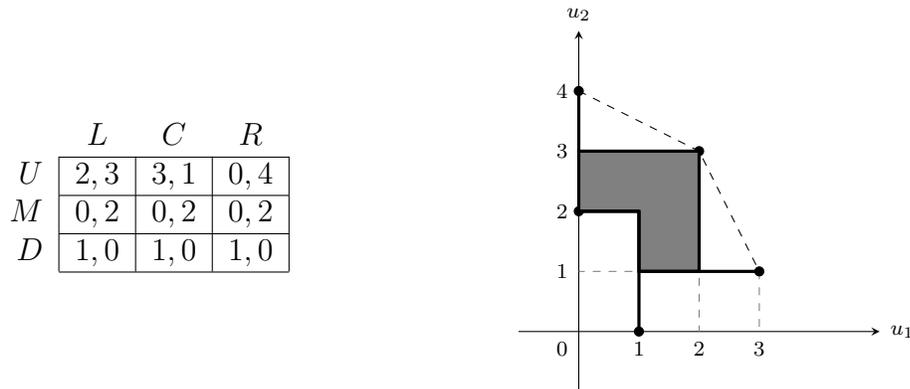


Figure B.3: The SPE payoff set under stochastic announcements under asynchronous proposer rule and limited specifiability: the payoff matrix (left) and the SPE payoff set of the negotiation game under stochastic announcements (right).

players play as in the pure SPE that supports it. □

The extension to stochastic announcements enables us to formalize the intuition of Farrell and Rabin (1996). They, in the context of a Battle of the Sexes, question the plausibility of an outcome that assigns equal probability to the two strict Nash equilibria as a consequence of negotiation: “...these fair equilibria do not seem likely to emerge. More plausibly, each player will argue for his or her preferred equilibrium.” We can formalize this intuition: When the component game is a Battle of the Sexes (with sufficiently low disagreement payoffs), such a “fair” outcome cannot be achieved under an asynchronous proposer rule and the consensual termination rule. Indeed, only equilibrium outcomes are the two strict Nash equilibria.⁴³

We can also establish the robustness of some characterizations of SPE sets. Namely, if U is convex, then X^M continues to be the set of SPE outcomes under asynchronous proposer rule and unlimited specifiability, and $IR(X, d)$ continues to be the set of SPE outcomes under synchronous proposer rule under any specifiability condition.

B.6 Various Negotiation Protocols

Our negotiation model is general and enables one to conduct comparison between various negotiation protocols. To demonstrate the wide applicability of the framework

⁴³When the proposer rule is synchronous, on the other hand, such an outcome can be achieved using jointly controlled lotteries à la Aumann, Maschler, and Stearns (1968). Thus the SPE payoff set in this case is a convex hull of the weakly individually rational alternatives. This difference is parallel to the difference between polite and non-polite talks in Aumann and Hart (2003).

and to guide the future work, here we provide some of possible rules of interest.

Termination rules

In the main analysis of this paper, we restricted attention to the consensual termination rule. One can vary termination rules to examine how such variations change the set of SPE outcomes.

1. *Coalitional consensual rules.* Our consensual rule implicitly assumes unanimity because all players have to be ok with x to terminate the negotiation with x . One can alternatively consider a rule in which there is a set of winning coalitions $\mathcal{C} \subseteq 2^N$ such that the negotiation terminates at h if there is $C \in \mathcal{C}$ such that (i) all players in C are ok with x at their respective latest opportunity after the latest No and (ii) at least one player in C speaks at $t(h)$. Our consensual rule corresponds to the case with $\mathcal{C} = \{N\}$.
2. *Majority rule.* A simple majority rule can be expressed as a termination rule. For example, consider a termination rule that ignores the Yes/No responses and terminates the negotiation right after all players have had chances to move, with an alternative that is announced the greatest number of times (with some tie-breaking rule).
3. *Deadline.* One can also express existence of a deadline by tuning the termination rule. For example, one can construct a new rule φ^T such that $\varphi^T(h) = \varphi^{\text{con}}(h)$ if $t(h) \leq T$ and $\varphi^T(h) = \text{Continue}$ if $t(h) > T$. This means that under φ^T , any negotiation process that lasts more than T periods necessarily ends up in the disagreement outcome. We conjecture that the existence of a deadline further facilitates commitment power. As the simplest example, with two players under an asynchronous proposer rule and an unlimited specification rule, the second-last mover can obtain the best payoff in $\text{IR}(U, d)$.
4. *Other rules in the literature.* Many of the negotiation protocols considered in the literature can be expressed as a subcase of our termination rule. For example, one can consider a termination rule that does not depend in any way on the Yes/No responses. Such a termination rule can be thought of as representing the protocol under which each party only announces their own proposals/actions.

As a concrete example, we formalize Bhaskar’s (1989) “quick response game” in Appendix B.9.

5. *k-consensual rules*. One can consider a class of termination rules such that a -necessary- condition to terminate is that every player is ok with the given outcome. A special case of this class is our consensual rule. In general, we can consider a termination rule that terminates with an alternative x at history h if at each of the latest k opportunities for each player i after the latest No, i is ok with x .
6. *An alternative definition of being ok*. We could also define a termination rule in a way that player i is ok with x at h when it is a unique element of the intersection of players’ proposals after her own most recent announcement of No, instead of the most recent announcement of No that she has observed or made. We provide an example to illustrate the difference between the consensual and the alternative termination rule in the Appendix B.7.

Stochastic rules

The main part of the paper dealt with a deterministic negotiation protocol and pure strategies. We considered the possibility of behavioral strategies in Section B.5. Here we consider the possibility of stochastic protocols.

1. *Stochastic proposer rules*. There are various bargaining models in which the set of proposers are randomly chosen each period (pioneered by Baron and Ferejohn (1989)). One could make the proposer rule stochastic to nest such random proposer models. Also, in some of bargaining models and revision games, the moving player is stochastically determined according to Poisson processes. We can approximate such a process by considering a stochastic proposer rule that allows for the empty set of proposers (which we do allow for in our main analysis).
2. *Stochastic specification rules*. In general we can consider stochastic specification rules. This may represent the situation where, for example, an interest group imposing a feasibility constraint on available proposals becomes conciliatory and such an event happens at random times (in the eyes of the negotiation parties).

3. *Stochastic termination rules.* One example of stochastic termination rules is that the negotiation ends for an exogenous reason with probability p each period. Upon ending with such a reason, there are various possibilities for the resulting outcome. One possibility is that the disagreement outcome results, and formally the termination rule returns “Continue” for any history after the exogenous ending. Another possibility is that the outcome x results, where x is the alternative with which the number of players who were ok is the greatest (with some tie-breaking rule). Yet another possibility is that the negotiation ends with probability p at each history at which every player is ok with some alternative. This last possibility is again in the class of termination rule such that a -necessary- condition to terminate is that every player is ok with the terminating outcome.

Other Possibilities

1. *Changing component games and side-payments.* It would be interesting to examine the effect of history-dependent negotiation protocols. Such extensions may allow for modeling players’ incentives to change the component game and to promise side-payments contingent on taking some alternatives.⁴⁴ We can do so by expanding X by having the state (the component game to be played at the moment) and/or the side-payment as part of the description of each alternative and by making available proposals dependent on histories.
2. *Agreeing on a subset of alternatives.* It would also be interesting to consider the possibility that players can agree on a subset of X (not just on a *single* alternative), and alternatives resulting from such agreements are exogenously or endogenously specified. If such alternatives are specified exogenously,⁴⁵ then one can model such a negotiation by tuning the termination rule. On the other hand, our model may not nest the endogenous case: Suppose that the component game corresponds to a normal-form game and specification rule corresponds to announcing a subset of the own action space. As in Renou (2009), one could consider an extension of our game with the possibility of agreements on subsets of X , where after a subset (a product set, which corresponds to a smaller game

⁴⁴See Jackson and Wilkie (2005) for a related model with side-payments.

⁴⁵That is, there exists a pre-specified mapping from 2^X to X .

than the original) is chosen, players play the normal-form game that corresponds to that subset. In that way, we could replace the commitment stage of Renou's game with a realistic negotiation phase, and examine the effect of the detail of such a negotiation phase on the SPE outcome of the whole game.

3. *Mediator*. Many real negotiations are conducted in the presence of a mediator. One way to add a mediator in our model would be to have a set of players $N \cup \{m\}$ where m represents the mediator. The mediator would not announce a response Yes or No but announce only a proposal. Her specification rule is unlimited. One reasonable preferences of the mediator would be that she prefers an alternative to another if the former Pareto dominates the latter for players in N . We would modify the consensual termination rule so that m would not need to be ok with an outcome but all other players in N would have to be.

B.7 Illustration of an Alternative Termination Rule

Consider the following component game, where the set of players is $N = \{1, 2, 3\}$ and the set of alternatives is $X = \{a, b, c\}$. We consider the unlimited specification rule. Consider the following history:

$$h = ((3, \text{Yes}, \{a, c\}), (2, \text{No}, \{b\}), (1, \text{Yes}, \{a, b\}), (3, \text{Yes}, \{b\})).$$

Note that $(3, \text{Yes}, \{a, c\})$, for example, denotes that player 3's proposal is $(\text{Yes}, \{a, c\})$. Player 3 starts proposing $(\text{Yes}, \{a, c\})$. Player 2 then replies with $(\text{No}, \{b\})$. In response to that, player 1 proposes $(\text{Yes}, \{a, b\})$. Then, player 3 replies with $(\text{Yes}, \{b\})$.

We compare the two termination rules with regards to this history. The one is the consensual termination rule defined in the main text. The other is its variant where player i is ok with x at a given history when the intersection of players' proposals after her own most recent announcement of No, instead of the most recent announcement of No that she has seen or made.

We demonstrate that this history is not a terminal history under the alternative termination rule, while it is a terminal history under the consensual termination rule. To see this, consider first the alternative termination rule. At the history h , player 3 announces $(\text{Yes}, \{b\})$. Since player 3's most recent announcement of No is made at time 0 and the intersection of players' most recent proposals after time 0 is

$\{b\} = \{b\} \cap \{a, b\} \cap \{b\}$, player 3 is ok with b at h under the alternative termination rule. Under the consensual termination rule, on the other hand, we consider the most recent announcement of No that player 3 has seen or made. Thus, we consider players' proposals after the history h^2 when player 2 says No. In this case, since the intersection of players' proposals also becomes $\{b\} = \{b\} \cap \{a, b\} \cap \{b\}$, player 3 is ok with b at h under the consensual termination rule.

Next, consider the history h^3 , after which player 1 speaks. At h^3 , since player 1's most recent announcement is made at time 0, the intersection of player's most recent proposals at h^3 in question becomes $\emptyset = \{a, b\} \cap \{b\} \cap \{a, c\}$ under the alternative termination rule. Hence, player 1 is *not* ok with any alternative at h^3 under the alternative termination rule. Under the alternative termination rule, we assume that player 1 takes player 3's announcement $\{a, c\}$ at h^1 into account even though player 2 has announced No at h^2 , because we define player i being ok with an alternative in terms of the proposals that have been made after player i 's her own most recent announcement of No. Under the consensual termination rule, on the other hand, player 1 is ok with $\{b\}$, the intersection of player 1's own proposal $\{a, b\}$ and player 2's proposal $\{b\}$ (precisely, note that player 3's relevant proposal is the entire set X), given that player 2 has announced No at h^2 .

At the history h^2 , player 2 is ok with b under both termination rules. Since player 2 herself announces No at that history, we consider only player 2's proposal at h^2 under both termination rules. Since her announcement $\{b\}$ is a singleton set, player 2 is ok with b under both termination rules.

Now, at h , the latest time at which some player announces No is 2(= $t^{\text{No}}(h)$) when player 2 says No. Every player's most recent response was made after that history, and every player is ok with the alternative b at respective history under the consensual termination rule. Hence, h is a terminal history under the consensual termination rule, and the outcome of the negotiation is $b = \varphi^{\text{con}}(h)$. Under the alternative termination rule, on the other hand, h is not a terminal history, because player 1 is not ok with any alternative at h^3 .

In our paper, we adopt the consensual termination rule, and analyze the effect of moves and specifiability structures on the negotiation outcome. We expect, however, that our main results would carry over to the case of the alternative termination rule as well.

B.8 The Case of $\text{IR}(U, d) = \emptyset$

If the component game G does not possess any weakly individually rational alternative, then the disagreement outcome is a unique (pure strategy) SPE outcome given that each player can announce the empty set.

Proposition B.3. *Consider a negotiation $\langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{con}} \rangle$ such that $\text{IR}(U, d) = \emptyset$ and that $\emptyset \in \mathcal{P}_i$ for every $i \in N$. Then the disagreement outcome is a unique SPE outcome.*

Proof of Proposition B.3. By Proposition 1, any alternative $x \in X$ cannot be a SPE outcome. Thus, it suffices to show that the disagreement outcome can be sustained as a SPE outcome.

In order to construct a SPE strategy profile, we first decompose, for each player $i \in N$, the set H_i into the following two subsets Q_k^i ($k \in \{1, 2\}$). Recall that H_i is the set of histories at which it is i 's turn to speak.

$$Q_1^i := \left\{ h \in H_i \mid \begin{array}{l} \text{there is a sequence } ((N_k, (\tilde{P}_j)_{j \in N_k}))_{k=1}^{k^*} \text{ such that } N_1 = \rho(h), \\ N_{\ell+1} = \rho(h, ((N_k, ((\text{Yes}, \tilde{P}_j))_{j \in N_k}))_{k=1}^{\ell}) \text{ for each } \ell \in \{1, \dots, k^* - 1\} \text{ (if } k^* \geq 2), \\ \varphi^{\text{con}}(h, (((\text{Yes}, \tilde{P}_j))_{j \in N_k})_{k=1}^{k^*}) = \tilde{x} \in X \setminus \{\hat{x}\}, \\ \text{and } u_j(\tilde{x}) > d_j \text{ for all } j \in \bigcup_{k=1}^{k^*} N_k \end{array} \right\}; \text{ and}$$

$$Q_2^i := H_i \setminus Q_1^i.$$

The set Q_1^i contains any non-terminal history h in H_i with the following properties: (i) some players have already been ok with an alternative \tilde{x} at h ;⁴⁶ and (ii) there is a sequence of players and proposals such that it is of each corresponding player's best interest to agree on \tilde{x} . Note that the set Q_1^i might be empty, depending on the component game. For example, if the component game does not possess $\tilde{x} \in X$ with $u_i(\tilde{x}) > d_i$, the set Q_1^i is empty.

⁴⁶Note that the alternative \tilde{x} might not be weakly individually rational for these players; indeed, there is at least one player who is ok with \tilde{x} at h and whose payoff is worse than her disagreement payoff.

We define players' strategy profile $s^* := (s_i^*)_{i \in N}$ as follows: At any history $h \in H_i$,

$$s_i^*(h) := \begin{cases} (\text{Yes}, \tilde{P}_i) & \text{if } h \in Q_1^i \\ (\text{No}, \emptyset) & \text{if } h \in Q_2^i \end{cases},$$

where $s_i^*(h) = (\text{Yes}, \tilde{P}_i)$ ($h \in Q_1^i$) is chosen so that \tilde{P}_i is consistent with a sequence $((N_k, (\tilde{P}_j)_{j \in N_k})_{k=1}^{k^*})$ such that $\varphi^{\text{con}}(h, (((\text{Yes}, \tilde{P}_j))_{j \in N_k})_{k=1}^{k^*}) = \tilde{x} \in X \setminus \{\hat{x}\}$. That is, for any given sequence $((N_k, (\tilde{P}_j)_{j \in N_k})_{k=1}^{k^*})$ such that $\varphi^{\text{con}}(h, (((\text{Yes}, \tilde{P}_j))_{j \in N_k})_{k=1}^{k^*}) = \tilde{x} \in X \setminus \{\hat{x}\}$, the set \tilde{P}_i is chosen from the sequence, where $i \in N_1$. Observe that since $u(\tilde{x}) \in \text{IR}(U, d)$, for any choice of such sequence $((N_k, (\tilde{P}_i)_{i \in N_k})_{k=1}^{k^*})$, the set $N \setminus \left(\bigcup_{k=1}^{k^*} N_k\right)$ is not empty. The player(s) in this set have to be ok with \tilde{x} at h , and hence for any choice of such a sequence, the consensual rule uniquely returns $\varphi^{\text{con}}(h, (((\text{Yes}, \tilde{P}_i))_{i \in N_k})_{k=1}^{k^*}) = \tilde{x} \in X$.

We show that, for each player $i \in N$, s_i^* is a best response to $s_{-i}^*(= (s_j^*)_{j \in N \setminus \{i\}})$ in any subgame. We find below the maximum possible payoff that player $i \in N$ can obtain in any subgame, given that every player $j \in N \setminus \{i\}$ follows s_j^* . We are going to show the following two statements.

1. In any subgame starting from $h \in H_i \setminus Q_1^i$, the maximum possible payoff that player i can obtain against s_{-i}^* is d_i .
2. In any subgame starting from $h \in Q_1^i$, then the maximum payoff that player i can obtain against s_{-i}^* in the subgame starting after such a history h is $u_i(\tilde{x}) > d_i$, where observe that \tilde{x} is uniquely determined by h .

Now, we prove the above statements. If the component game does not have an alternative $\tilde{x} \in X$ with $u_i(\tilde{x}) > d_i$, then the above two statements are trivially true (player i always announces (No, \emptyset) at each history at which it is her turn to speak, inducing the disagreement outcome). Hence, suppose that the component game contains an alternative $\tilde{x} \in X$ with $u_i(\tilde{x}) > d_i$.

First, consider any subgame starting from $h \in H_i \setminus Q_1^i$. If player i 's announcement induces the history $h' = (h, (P_i, R_i)) \notin Q_1^j$ (it is without loss of generality to assume $\rho(h') \neq \emptyset$) for some $j \in \rho(h')$, every player $k \in N \setminus \{i\}$ keeps announcing (No, \emptyset) at any history at which j speaks in the subgame after h^{t+2} (irrespective of player i 's possible opportunities to speak after h and h'). Hence, the disagreement outcome is induced after h^{t+1} . If, on the other hand, player i 's announcement induces the history

$h^{t+1} = (h, (P_i, R_i))$ with $h' \in Q_1^j$ (it is without loss of generality to assume $\rho(h') \neq \emptyset$) for all $j \in \rho(h')$, either the disagreement outcome is induced or some alternative which is not weakly individually rational for player i is induced. Following s_i^* induces the disagreement outcome, and hence in any subgame starting from $h \in H_i \setminus Q_1^i$, following s_i^* is a best response to s_{-i}^* .

Second, we show that in any subgame starting from $h \in Q_1^i$, the maximum payoff that player i can obtain against s_{-i}^* is $u_i(\tilde{x}) (> d_i)$. Consider any subgame starting from such a history h . If player i chooses to follow s_i^* in the subgame, then the strategy profile s^* induces the outcome \tilde{x} . As long as player i 's announcement at each history h' at which it is her turn to speak in the subgame starting after h is consistent with a sequence $((N_k, (\tilde{P}_j)_{j \in N_k})_{k=1}^{k^*})$ such that $\varphi^{\text{con}}(h', ((\text{Yes}, \tilde{P}_j)_{j \in N_k})_{k=1}^{k^*}) = \tilde{x} \in X$ (that is, for any given sequence $((N_k, (\tilde{P}_j)_{j \in N_k})_{k=1}^{k^*})$ such that $\varphi^{\text{con}}(h', ((\text{Yes}, \tilde{P}_j)_{j \in N_k})_{k=1}^{k^*}) = \tilde{x}$, her announcement \tilde{P}_i is chosen from the sequence, where $i \in N_1$), the strategy profile induces the outcome \tilde{x} .

Suppose, on the other hand, player i 's announcement at a history h' at which it is her turn to speak in the subgame starting after h induces a history h'' , where there is no sequence $((N_k, (\tilde{P}_j)_{j \in N_k})_{k=1}^{k^*})$ such that $N_1 = \rho(h'')$, $N_{\ell+1} = \rho(h'', ((N_k, ((\text{Yes}, \tilde{P}_j)_{j \in N_k}))_{k=1}^{\ell}))$ for each $\ell \in \{1, \dots, k^* - 1\}$ (if $k^* \geq 2$), $\varphi^{\text{con}}(h'', ((\text{Yes}, \tilde{P}_j)_{j \in N_k})_{k=1}^{k^*}) = \tilde{x}$, and $u_j(\tilde{x}) > d_j$ for all $j \in \bigcup_{k=1}^{k^*} N_k$. Then, every player $j \in N \setminus \{i\}$ keeps announcing (No, \emptyset) at any history at which j speaks in the subgame starting after h'' . Hence, only the disagreement outcome is induced after h'' . Thus, the maximum payoff that player i can obtain against s_{-i}^* in the subgame following h is $u_i(\tilde{x})$.

Hence, s_i^* is a best response to s_{-i}^* in any subgame, and hence the strategy profile s^* is a SPE. The pair s^* induces the terminal history which is associated with the disagreement outcome. \square

B.9 Quick Response Games

Here we formulate Bhaskar's (1989) termination rule in our framework and study its implication on negotiation outcomes.

Formulation

We suppose that a component game G is induced from a two-player normal-form game $\langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$ with $X = A$. The "quick-response" game $\Gamma = \langle G, d, \rho, (\mathcal{P}_i)_{i \in N}, \varphi^{\text{QR}} \rangle$

	<i>L</i>	<i>R</i>
<i>U</i>	4, 2	0, 0
<i>D</i>	0, 0	2, 4

Table B.1: Battle of the Sexes Game

of G can be expressed as a variant of our negotiation game as follows. The disagreement payoff $d \in \mathbb{R}^2$ satisfies that $\text{IR}(U, d) \neq \emptyset$. The proposer rule is asynchronous: Player 1 moves in odd periods and player 2 moves in even periods. Formally, $\rho(h) = i$ for any h , where $i - 1$ is the remainder of $t(h)$ by 2. Players can only announce their actions, that is, $\mathcal{P}_i = \{\{a_i\} \times A_{-i} \mid a_i \in A_i\}$ for each $i \in N$. Finally, the quick-response termination rule $\varphi^{\text{QR}} : \mathcal{H} \rightarrow A \cup \{\text{Continue}\}$ is given as follows.

$$\varphi^{\text{QR}}(h) = \begin{cases} (a_1, a_2) & \text{if } h = (h^{t(h)-3}, (\cdot, a_i), (\cdot, a_{-i}), (\cdot, a_i)) \\ \text{Continue} & \text{otherwise} \end{cases}.$$

The quick-response termination rule φ^{QR} terminates the game whenever some player i has announces the same action a_i twice in a row. Since The quick-response termination rule φ^{QR} does not depend on players' Yes/No responses, we henceforth omit the reference to Yes/No responses when describing actions.

While we think the consensual termination rule to be natural in negotiations where a consensus requires parties' unanimous agreement, the quick-response termination rule may serve as a termination rule in such a situation as Bhaskar's (1989) duopoly game, where explicit communication between players is absent. Our framework can accommodate such a negotiation game as well.

Before we analyze some examples of quick response games in our framework, we remark that the solution concepts between Bhaskar's (1989) and our models are different. We characterize the SPE payoffs under the quick-response games, while Bhaskar (1989) demonstrates that a monopoly price can be sustained as a unique equilibrium outcome when the equilibrium strategies are required so that they are not weakly dominated after any history.

Examples

Under the quick-response termination rule, we have the following results. First, as in our model with the consensual rule, a unique Pareto-efficient alternative can be sustained as a SPE when there is a common-interest alternative. Second, the quick response game exhibits the second-mover advantage when a component game is a Battle of the Sexes as in Table B.1.⁴⁷ We consider this second example in detail.

In what follows, we refer to the quick-response game as the quick-response game of the Battle of the Sexes game depicted in Table B.1. In Proposition B.4, we show that the unique SPE outcome of the quick-response game is (D, R) . The proof of Proposition B.4 follows from the following two lemmas. First, Lemma B.1 demonstrates that the quick-response game has a SPE. Second, we show an intermediate result examining the properties of the SPE of this quick response game.

Lemma B.1. *The quick-response game of the Battle of the Sexes has a SPE.*

Lemma B.2. *Let $s = (s_i)_{i \in N}$ be a SPE of the quick-response game of the Battle of the Sexes. Then, player 1's strategy s_1 satisfies the following: For each $h \in H_1$,*

$$s_1(h) = \begin{cases} D & \text{if } h = (h^{t(h)-2}, U, R) \\ U & \text{if } h \in \{(h^{t(h)-2}, D, R), (h^{2t-1}, D, L)\} \end{cases}.$$

Player 2's strategy satisfies the following: For each $h \in H_2$,

$$s_2(h) = \begin{cases} L & \text{if } h \in \{(D), (h^{t(h)-2}, R, U)\} \\ R & \text{if } h \in \{(U), (h^{t(h)-2}, L, U), (h^{2t-1}, L, D)\} \end{cases}.$$

Proposition B.4. *The unique equilibrium outcome of the quick-response game of the Battle of the Sexes is (D, R) .*

Proof of Proposition B.4. Let s be a SPE of the quick-response game. If $s_1(h^0) = U$, then it follows immediately from Lemma B.2 that s induces the history

$$h = (s_1(h^0), s_2(s_1(h^0)), s_1(s_1(h^0), s_2(s_1(h^0)))) = (U, R, D).$$

⁴⁷A component game exhibits the second-mover advantage as long as the following are satisfied: (i) (U, L) and (D, R) are strict Nash equilibria; (ii) player 1 prefers (U, L) to (D, R) ; player 2 prefers (D, R) to (U, L) ; (iii) each player still prefers her/his less-preferred Nash equilibrium outcome to non-equilibrium outcomes; and (iv) each player's preferred Nash equilibrium outcome is better than the disagreement.

Then, the outcome after the history h must be (D, R) , since player 2 can obtain his maximum payoff $u_2(D, R)$ by choosing R after h and terminating the game.

If $s_1(h^0) = D$, then it follows immediately from Lemma B.2 that s induces the history $h' = (D, L, U, R, D)$ up to period 5. Then, the outcome after the history h' must be (D, R) , since player 2 can obtain his maximum payoff $u_2(D, R)$ by choosing R after h and terminating the game. The proof is complete. \square

Proof of Lemma B.1. The following strategy profile $s^* = (s_i^*)_{i \in N}$ is a SPE of the quick-response game which sustains the outcome (D, R) . Player 1's strategy is defined as follows: At the initial history h^0 , $s_1^*(h^0) = U$. For each $h \in H_1$,

$$s_1^*(h) = \begin{cases} D & \text{if } h = (h^{t(h)-2}, U, R) \\ U & \text{otherwise} \end{cases}.$$

Likewise, player 2's strategy is defined as follows: At a history $h \in H_2$,

$$s_2^*(h) = \begin{cases} L & \text{if } h \in \{(D), (h^{t(h)-2}, R, U)\} \\ R & \text{otherwise} \end{cases}.$$

We show that s^* constitute a SPE. We show that player $i (\in N)$ following s_i^* is a best response to s_{-i}^* in any subgame. In order to simplify the notation, hereafter, we drop the Yes/No responses from histories and strategies.

First, consider the subgame starting after a history $h = (h^{t(h)-2}, R, D)$. Following s_2^* is a best response to s_1^* in the subgame, since player 2 can obtain his maximum payoff $u_2(D, R)$ in period $t(h) + 1$ by playing $s_2^*(h) = R$ and terminating the game.

Consider the subgame starting after a history $h = (h^{t(h)-2}, U, L)$. Following s_1^* is a best response to s_2^* in the subgame, since player 1 can obtain her maximum payoff $u_1(U, L)$ in period $t(h) + 1$ by playing $s_1^*(h) = U$ and terminating the game.

Consider the subgame starting after a history $h = (h^{t(h)-2}, R, U)$. We show that following s_2^* is a best response to s_1^* in the subgame. If player 2 chooses $s_2^*(h) = L$ after h , then player 1, who follows the strategy s_1^* , chooses $s_1^*(h^{t(h)-1}, U, L) = U$ and the game ends. Player 2 obtains the payoff of $u_2(U, L)$. If player 2 chooses R after h , then the game immediately ends, and he obtains the payoff of $u_2(U, R) < u_2(U, L)$. Hence, following s_2^* is a best response to s_1^* in the subgame.

Consider the subgame starting after a history $h = (h^{t(h)-2}, U, R)$. We show that

following s_1^* is a best response to s_2^* in the subgame. If player 1 chooses $s_1^*(h) = D$ after h , then player 2, who follows the strategy s_2^* , chooses $s_2^*(h^{t(h)-1}, R, D) = R$ and the game ends. Player 1 obtains the payoff of $u_1(D, R)$. If player 1 chooses U , then the game immediately ends, and she obtains the payoff of $u_1(U, R) < u_2(D, R)$. Hence, following s_1^* is a best response to s_2^* in the subgame.

Consider the subgame starting after a history $h = (h^{t(h)-2}, L, U)$. We show that following s_2^* is a best response to s_1^* in the subgame. If player 2 chooses $s_2^*(h) = R$ after h , then player 1, who follows the strategy s_1^* , chooses D . Then, player 2 can obtain his maximum payoff $u_2(D, R)$ by playing $s_2^*(h, R, D) = R$ after (h, R, D) . Hence, playing s_2^* is a best response to s_1^* in the subgame.

Consider the subgame starting after a history $h = (h^{t(h)-2}, D, R)$. We show that playing s_1^* is a best response to s_2^* in the subgame. If player 1 chooses $s_1^*(h) = U$ after h , then player 2, who follows the strategy s_2^* , chooses L . Then, player 1 can obtain her maximum payoff $u_1(U, L)$ by playing $s_1^*(h, U, L) = U$ after (h^{2t}, U, L) . Hence, playing s_1^* is a best response to s_2^* in the subgame.

Consider the subgame starting after a history $h = (h^{t(h)-2}, L, D)$. We show that playing s_2^* is a best response to s_1^* in the subgame. If player 2 chooses L , then the game immediately ends, and he obtains a payoff of $u_2(D, L)$. Suppose now that player 2 chooses $s_2^*(h) = R$ after h . Then, player 1, who follows the strategy s_1^* , chooses U . The resulting history is $h' = (h^{t(h)-1}, L, D, R, U)$. Now, if player 2 plays $s_2^*(h') = L$, then player 1 plays $s_1^*(h', L) = U$ after (h', L) , and the game ends. Player 2 obtains a payoff of $u_2(U, L)$. If, on the other hand, player 2 chooses R after (h, R, U) , then the game ends and he receives a payoff of $u_2(U, R)$. Since $u_2(U, L) > \max\{u_2(U, R), u_2(D, L)\}$, following s_2^* is a best response to s_1^* in the subgame starting from such a history h .

Consider the subgame starting after a history $h = (h^{t(h)-2}, D, L)$. We show that playing s_1^* is a best response to s_2^* in the subgame. If player 1 chooses D , then the game immediately ends, and she obtains a payoff of $u_1(D, L)$. Suppose now that player 1 chooses $s_1^*(h) = U$ after h . Then, player 2, who follows the strategy s_2^* , chooses $s_2^*(h, U) = R$. The resulting history is (h, U, R) . If player 1 chooses $s_1^*(h, U, R) = D$, then player 2 chooses $s_2^*(h, U, R, D) = R$ and the game ends. Player 1's payoff is $u_1(D, R)$. If, on the other hand, player 1 chooses U after (h, U, R) , then the game immediately ends and she receives the payoff $u_1(U, R)$. Since $u_1(D, R) > \max\{u_1(U, R), u_1(D, L)\}$, following s_1^* is a best response to s_2^* in the subgame starting from such a history h^{2t} .

Consider the subgame starting after the history $h = (U)$. Thus, $\rho(h) = 2$. After the history $h = (U)$, if player 2 chooses $s_2^*(U) = R$, then player 1, who follows the strategy s_1^* , chooses $s_1^*(U, R) = D$. Then, player 2 can obtain his maximum payoff $u_2(D, R)$ by playing $s_2^*(U, R, D) = R$ after the history (U, R, D) . Hence, playing s_2^* is a best response to s_1^* in the subgame.

Consider the subgame starting after the history $h = (D)$. Thus, $\rho(h) = 2$. After the history $h = (D)$, if player 2 chooses $s_2^*(D) = L$, then player 1, who follows the strategy s_1^* , chooses U after (D, L) . Then, player 2 can obtain his maximum payoff $u_2(D, R)$ by following s_2^* after (D, L, U) : By playing $s_2^*(D, L, U) = R$, the history (D, L, U, R) is generated. Then, player 1 chooses $s_1^*(D, L, U, R) = D$. Now, player 2 can obtain a payoff of $u_2(D, R)$ by choosing $s_2^*(D, L, U, R, D) = R$. Hence, playing s_2^* is a best response to s_1^* in the subgame.

Finally, consider the subgame starting after the initial history h^0 . Thus, $\rho(h^0) = 1$. We show that the maximum payoff that player 1 can obtain against the strategy s_2^* in the subgame starting the initial history is $u_1(D, R)$. First, it is easily seen that s^* generates the history (U, R, D, R) and the outcome (D, R) .

After the initial history h^0 , suppose that player 1 chooses $s_1^*(h^0) = U$. Then, player 2 chooses $s_2^*(U) = R$, yielding the history (U, R) . If player 1 chooses U after (U, R) , then the game ends, and she receives a payoff of $u_1(U, R)$. If player 1 chooses $s_1^*(U, R) = D$ after (U, R) , then player 2 chooses $s_2^*(U, R, D) = R$ after $h^3 = (U, R, D)$, and the game ends. Player 1 receives the payoff of $u_1(D, R)$.

Now, suppose that player 1 chooses D after the initial history. Then, player 2 chooses $s_2^*(D) = L$. If player 1 chooses D after the history (D, L) , then the game ends, and she receives a payoff of $u_1(D, L)$. If player 1 chooses U after the history (D, L) , then player 2 chooses $s_2^*(D, L, U) = R$. If player 1 chooses U after the history (D, L, U, R) , then the game ends and she obtains a payoff of $u_1(U, R)$. If player 1 chooses D after the history (D, L, U, R) , then player 2 chooses $s_2^*(D, L, U, R, D) = R$, and the game ends. Player 1 receives a payoff of $u_1(D, R)$.

Hence, the possible histories when player 2 follows s_2^* are as follows: (U, R, U) , (U, R, D, R) , (D, L, D) , (D, L, U, R, U) , and (D, L, U, R, D, R) . The maximum payoff that player 1 can obtain against s_2^* in the subgame starting from the initial history is $u_1(D, R)$, obtained from the histories (U, R, D, R) or (D, L, U, R, D, R) . If player 1 follows the strategy s_1^* , then the strategy profile s^* generates the history (U, R, D, R) . Hence, playing s_1^* is a best response.

In sum, the strategy profile s^* constitutes a SPE of the quick-response game. \square

Proof of Lemma B.2. In order to simplify the notation, we drop the Yes/No responses from histories and strategies. Consider the subgame starting after a history $h = (h^{t(h)-2}, R, D)$. In any SPE, the outcome after such a history h must be (D, R) , since player 2 can obtain his maximum payoff $u_2(D, R)$ by announcing R after h and terminating the game.

Consider the subgame starting after a history $h = (h^{t(h)-2}, U, L)$. In any SPE, the outcome after such a history h must be (U, L) , since player 1 can obtain her maximum payoff $u_1(U, L)$ by announcing U after h and terminating the game.

Consider the subgame starting after a history $h = (h^{t(h)-2}, R, U)$. We show that in any SPE s , player 2 plays $s_2(h^{t(h)-2}, R, U) = L$ and the outcome after such a history h must be (U, L) . If player 2 chooses L , then the resulting history is (h, L) , and it follows from the above observation that the outcome is (U, L) . If player 2 chooses R , then the game ends and the outcome is (U, R) . Since $u_2(U, L) > u_2(U, R)$, player 2 chooses $s_2(h^{t(h)-2}, R, U) = L$.

Consider the subgame starting after a history $h = (h^{t(h)-2}, U, R)$. We show that in any SPE s , player 1 plays $s_1(h^{t(h)-2}, U, R) = D$ and the outcome after such a history h must be (D, R) . If player 1 chooses D , then the resulting history is (h, D) , and it follows from the above observation that the outcome is (D, R) . If player 1 chooses U , then the game ends and the outcome is (U, R) . Since $u_1(D, R) > u_1(U, R)$, player 1 chooses $s_1(h) = D$.

Consider the subgame starting after a history $h = (h^{t(h)-2}, L, U)$. We show that in any SPE s , player 2 plays $s_2(h) = R$ and the outcome after such a history (h, U) must be (D, R) . If player 2 chooses R , then the resulting history is (h, U, R) , and it follows that the outcome is (D, R) . If player 2 plays L after (h, U) , then the game ends with the outcome (U, L) . Player 2 obtains a payoff of $u_2(U, L) < u_2(D, R)$. Thus, in any SPE $s = (s_i)_{i \in N}$, player 2 plays $s_2(h, U) = R$ and the outcome after such a history (h, U) is (D, R) .

Consider the subgame starting after a history $h = (h^{t(h)-2}, D, R)$. We show that in any SPE s , player 1 plays $s_1(h) = U$ and the outcome after such a history h must be (U, L) . If player 1 chooses U , then the resulting history is (h, U) , and it follows that the outcome is (U, L) . If player 1 plays D after h , then the game ends with the outcome (D, R) . Player 1 obtains a payoff of $u_1(D, R) < u_1(U, L)$. Thus, in any SPE $s = (s_i)_{i \in N}$, player 1 plays $s_1(h) = U$ and the outcome after such a history h is (U, L) .

Consider the subgame starting after a history $h = (h^{t(h)-2}, L, D)$. We show that in any SPE s , player 2 plays $s_2(h^{t(h)-2}, L, D) = R$ and the outcome after such a history (h, D) must be (U, L) . If player 2 chooses R , then the resulting history is (h, R) , and it follows that the outcome is (U, L) . If player 2 chooses L , then the game ends and the outcome is (D, L) . Since $u_2(U, L) > u_2(D, L)$, player 2 chooses $s_2(h) = R$ after (h, R) , and the outcome after (h, R) is (U, L) .

Consider the subgame starting after a history $h = (h^{t(h)-2}, D, L)$. We show that in any SPE s , player 1 plays $s_1(h) = U$ and the outcome after such a history h must be (D, R) . If player 1 chooses U , then the resulting history is (h, U) , and it follows that the outcome is (D, R) . If player 1 chooses D , then the game ends and the outcome is (D, L) . Since $u_1(D, R) > u_1(D, L)$, player 1 chooses $s_1(h) = U$, and the outcome after h is (D, R) .

Next, we show that in any SPE, after the history $h = (U)$, we have $s_2(h) = R$ and the outcome must be (D, R) . After the history h , playing L induces the history (U, L) . It follows that the final outcome must be (U, L) after the history (U, L) . If player 2 chooses R after the history h , then the resulting history is (U, R) . Then, it follows that the outcome must be (D, R) after the history (U, R) . Thus, player 2 chooses $s_2(h) = R$ after the history h in any SPE.

Finally, we show that in any SPE, after the history $h = (D)$, we have $s_2(h) = L$ and the outcome must be (D, R) . After the history h , playing L induces the history (D, L) . It follows that the final outcome must be (D, R) after the history (D, L) . If player 2 chooses R after the history h , then the resulting history is (D, R) . Then, it follows that the outcome must be (U, L) after the history (D, R) . Thus, player 2 chooses $s_2(h) = L$ after the history h in any SPE. \square

B.10 Checking the Best Response Condition for the Proof of Proposition 5

Step 1. We show that player $j \in N$ following s_j^* is a best response to s_{-j}^* in any subgame. First, we find the maximum possible payoff that player $j \in N$ can obtain in a subgame starting after each history, given that the other player $-j \in N \setminus \{j\}$ follows s_{-j}^* . Let $j = i$.

Case 1. In any subgame starting from $h \in H_i \cap (Q_0 \cup Q_1)$, the maximum possible payoff that player i can obtain against s_{-i}^* is $u_i(x) (\geq d_i)$.

Case 2. In any subgame starting from $h \in H_i \cap Q_2$, the maximum possible payoff that player i can obtain against s_{-i}^* is $u_i(x^{[i,0]}) (= v^{[i,M]})$.

Next, let $j = -i$.

Case 1'. In any subgame starting from $h \in H_{-i} \cap (Q_0 \cup Q_1)$, the maximum possible payoff that player $-i$ can obtain against s_i^* is $u_{-i}(x) (\geq d_{-i})$.

Case 2'. In any subgame starting from $h \in H_{-i} \cap Q_2$, the maximum possible payoff that player $-i$ can obtain against s_i^* is (i) $u_{-i}(\tilde{x})$ if $h = (h^{t(h)-2}, (R_{-i}, \tilde{P}_{-i}), (\text{Yes}, \tilde{P}_i))$ with $t(h) \geq 2$, $u_{-i}(\tilde{x}) > u_{-i}(x^{[i,0]})$, and $\{\tilde{x}\} = \tilde{P}_1 \cap \tilde{P}_2$; and (ii) $u_{-i}(x^{[i,0]})$ otherwise.

Consider $j = i$. First, Case 2 follows from the similar argument to Theorem 2. Now consider Case 1. Suppose any subgame starting from $h \in Q_0 \cup Q_1$ with $P(h) = i$. Player $(-i)$, who follows the strategy s_{-i}^* , always announces (No, P_{-i}) in response to $(R_i, P'_i) \in \{\text{Yes}, \text{No}\} \times (\mathcal{P}_i \setminus \{P_i, \emptyset\})$ and (Yes, P_{-i}) in response to $(R_i, P'_i) \in \{\text{Yes}, \text{No}\} \times \{P_i, \emptyset\}$. Thus, any induced non-terminal history h' is in $Q_0 \cap Q_1$ as long as player $-i$ follows the constructed strategy s_{-i}^* . Now, for any terminal history h' that induces an alternative as an outcome of the negotiation, player $-i$ has to be ok with x at either $h^{t(h')-1}(h)$ or h . Hence, x is the only alternative that can be an outcome of the negotiation game. Thus, in any subgame starting from $h \in Q_0 \cup Q_1$ with $P(h) = i$, the maximum possible payoff that player i can obtain against s_{-i}^* is $u_i(x) (\geq d_i)$.

Consider $j = -i$. First, Case 2 follows again from the arguments in Theorem 2. Suppose that in a subgame starting from $h \in Q_0 \cup Q_1$ with $P(h) = -i$, player $-i$ gets a payoff $u_{-i}(\tilde{x}) > u_{-i}(x)$. Suppose that a history h' is the associated terminal history (i.e., $\varphi^{\text{con}}(h') = \tilde{x}$).

Now, we must have either (i) a history of the form

$$h' = \left(h^{t(h')-3}, (R_{-i}, \tilde{P}_{-i}), (\text{Yes}, \tilde{P}_i), (\text{Yes}, \tilde{P}'_{-i}) \right)$$

with $t(h') \geq 3$, $u_{-i}(\tilde{x}) > u_{-i}(x) (\geq u_{-i}(x^{[i,0]}))$, and $\{\tilde{x}\} = \tilde{P}_i \cap \tilde{P}'_{-i} = \tilde{P}_i \cap \tilde{P}'_{-i}$, is generated after h ; or (ii) a history of the form

$$h' = \left(h^{t(h')-3}, (R_i, \tilde{P}_i), (\text{Yes}, \tilde{P}_{-i}), (\text{Yes}, \tilde{P}'_i) \right)$$

with $t(h') \geq 3$, $u_{-i}(\tilde{x}) > u_{-i}(x) (\geq u_{-i}(x^{[i,0]}))$, and $\{\tilde{x}\} = \tilde{P}_i \cap \tilde{P}_{-i} = \tilde{P}'_i \cap \tilde{P}_{-i}$, is generated after h .

Suppose that the case (i) happens. Then, since player i has to be ok with \tilde{x} , it must be the case that $\{\tilde{x}\} = \tilde{P}_i \cap \tilde{P}_{-i}$. If $h^{t(h')-2} \in Q_0 \cup Q_1$, it must be the case that $\tilde{P}_i = P_i$ and $\tilde{P}_{-i} = P_{-i}$. Thus, $\tilde{P}_i \cap \tilde{P}_{-i} = \{\tilde{x}\} \neq \{x\} = P_i \cap P_{-i}$, a contradiction. If $h^{t(h')-2} \in Q_2$, then it must be the case that $\tilde{P}_i = P_i^{(-i)}$ and $\tilde{P}_{-i} = P_{-i}^{(-i)}$. Thus, $\tilde{P}_i \cap \tilde{P}_{-i} = \{x^{[i,0]}\}$. Since $u_{-i}(x^{[i,0]}) \leq u_{-i}(x) < u_{-i}(\tilde{x})$, we have $x^{[i,0]} \neq x$, a contradiction.

Suppose that the case (ii) happens. If $h^{t(h')-2} \in Q_2$, then player $-i$ can get at most $u_{-i}(x^{[i,0]}) \leq u_{-i}(x)$, a contradiction. If $h^{t(h')-2} \in Q_0 \cup Q_1$, then it follows from the assumption that there is no P'_{-i} such that $\{x'\} = P_i \cap P'_{-i}$ and $u(x') > u(x)$. Hence, player $-i$'s proposal $P'_{-i} (\neq P_{-i})$ induces a history in Q_2 , after which player $-i$ can obtain at most $u_{-i}(x^{[i,0]}) \leq u_{-i}(x)$, a contradiction.

Now, we show that each player j can obtain her maximum possible payoff against s^*_{-j} by following s^*_j in each subgame. For each player $j \in N$, and for the case (2), the statements immediately follow from the proof of Theorem 2. For a subgame starting after $h \in Q_0 \cup Q_1$ with $P(h) = j$, the strategy profile s^* obviously induces the alternative x , as each player k keeps announcing (Yes, P_k) until the alternative x can be obtained as a final outcome.

Hence, s^*_j is a best response to s^*_{-j} in any subgame, and hence the strategy profile s^* is a SPE. The pair s^* induces the history $((\text{Yes}, P_1), (\text{Yes}, P_2), (\text{Yes}, P_1))$ and the outcome x .

Step 2.

We show that player $i \in N$ following s^*_i is a best response to s^*_{-i} in any subgame. We proceed by finding the maximum possible payoff that player $i \in N$ can obtain in a subgame starting after each history, given that the other player $-i \in N \setminus \{i\}$ follows s^*_{-i} . For player $i \in N$, in any subgame, the maximum payoff that player i can obtain against s^*_{-i} is given as follows.

Case 1. In any subgame starting from $h \in H_i \cap Q_0$, the maximum payoff that player i can obtain against s^*_{-i} is $u_i(x)$;

Case 2. In any subgame starting from $h \in H_i \cap Q_{-i}^{\text{on}}$, the maximum payoff that player i can obtain against s^*_{-i} is $u_i(x^{[-i,1]})$;

Case 3. In any subgame starting from $h \in H_i \cap Q_{-i}^{\text{off}}$, the maximum payoff that

player i can obtain against s_{-i}^* is (i) $u_i(\bar{x})$ if $h = (h^{t(h)-2}, (R_i, \bar{P}_i), (\text{Yes}, \bar{P}_{-i}))$, $t(h) \geq 2$, $\bar{P}_1 \cap \bar{P}_2 = \{\bar{x}\}$, and $u_i(\bar{x}) > u_i(x^{[-i,1]})$; and (ii) $u_i(x^{[-i,1]})$ otherwise.

For each player $i \in N$, and for each case (2)-(3), the statements immediately follow from Step 1. It also follows from Step 1 that the strategy profile s^* indeed induces the outcome that attains the maximum payoff that each player can obtain in each subgame. Thus, what remains is to characterize each player's maximum payoff in a subgame starting from any (non-terminal) subhistory of $h = ((\text{Yes}, P_1), (\text{Yes}, P_2), (\text{Yes}, P_1))$ (i.e., a history in Q_0).

Consider the subgame starting from $h^2 = ((\text{Yes}, P_1), (\text{Yes}, P_2))$. If player 1 chooses to announce $(R_1, P'_1) \in \{\text{Yes}, \text{No}\} \times \mathcal{P}_1 \setminus \{(\text{Yes}, P_1)\}$ after h^2 , then her maximum payoff in the subgame starting after $(h^2, (R_1, P'_1), s_2^*(h^2, (R_1, P'_1)))$ is $u_1(x^{[1,1]}) (\leq u_1(x))$. If player 1 chooses (Yes, P_1) after h^2 , then the game ends, yielding player 1 a payoff of $u_1(x) (\geq u_1(x^{[1,1]}))$.

Next, consider the subgame starting from the initial history h^0 . If player 1 chooses $(R_1, P'_1) \in \{\text{Yes}, \text{No}\} \times \mathcal{P}_1 \setminus \{(\text{Yes}, P_1)\}$ at the initial history, then her maximum payoff in the subgame starting after $((R_1, P'_1), s_2^*(R_1, P'_1))$ is $u_1(x^{[1,1]})$. Suppose that player 1 plays (Yes, P_1) . Then, player 2 chooses $s_2^*(\text{Yes}, P_1) = (\text{Yes}, P_2)$ after $h^1 = ((\text{Yes}, P_1))$, inducing the history h^2 . We have already shown that the maximum payoff that player 1 can obtain against s_2^* in the subgame starting after the history h^2 is $u_1(x)$. Hence, player 1's maximum payoff against s_2^* in the subgame starting from the initial history is $u_1(x)$.

Finally, consider the subgame starting from h^1 . If player 2 chooses $(R_2, P'_2) \in \{\text{Yes}, \text{No}\} \times \mathcal{P}_2 \setminus \{(\text{Yes}, P_2)\}$ at the history h^1 , then her maximum payoff in the subgame starting after $((\text{Yes}, P_1), (R_2, P'_2), s_1^*((\text{Yes}, P_1), (R_2, P'_2)))$ is $u_2(x^{[2,1]})$. Suppose that player 2 plays (Yes, P_2) . Then, player 1 chooses $s_1^*((\text{Yes}, P_1), (\text{Yes}, P_2)) = (\text{Yes}, P_1)$ after h^2 , and the game ends. Thus, we conclude that player 2's maximum payoff against s_1^* in the subgame starting from the history h^1 is $u_2(x)$.

Hence, s_i^* is a best response to s_{-i}^* in any subgame, and hence the strategy profile s^* is a SPE. The pair s^* induces the history $((\text{Yes}, P_1), (\text{Yes}, P_2), (\text{Yes}, P_1))$ and the outcome x . \square

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